DIGGING UP A PARADOX: A PHILOLOGICAL NOTE ON ZENO'S STADIUM

Of Zeno's four arguments[1]) against the reality of motion transmitted by Aristotle, the fourth, the so-called *Stadium* (*Vors.* 29 A 28), is perhaps the most difficult. The difficulties involved are of two sorts: philological problems on the one hand, questions of a philosophical nature on the other. In the present paper, I am concerned with the first sort, not the second, although I shall perhaps not be successful in keeping the latter out altogether. A study of the philosophical discussions to be found in the learned literature, however, has convinced me that the first problem to be solved is that of the interpretation of Aristotle's text. There is a general feeling that Aristotle, in reporting and arguing against Zeno's argument, somehow failed. I believe his report is sufficiently clear; although Aristotle's argument *contra* Zeno is not, perhaps, satisfactory in every respect, Zeno's original paradox can be found in his text. I shall attempt to show that, in order to find it, we must begin by taking both the topography of the stadium and the position of the bodies in it into account, which several recent reconstructions, however satisfactory they may appear to be in other respects, fail to do[2]).

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1) It is often said that, *pace* Aristotle, there were no more than four arguments against motion. But at *Phys.* Z 9, 239 b 9f. (*Vors.* 29 A25) Aristotle says there are four such arguments "which give trouble to those who try to solve the problems they involve" (tr. H.D.P. Lee, *Zeno of Elea,* Cambridge 1936, repr. Amsterdam 1967, 43). This may entail that there were others, which were believed to have been successfully refuted already; cf. Lee, *o. c.,* 66. However this may be, it is certainly wrong to separate the fourth argument from the other three, *pace* K. v. Fritz, *Zenon 1) von Elea,* in: *Pauly-Wissowa* Vol. XA (Munich 1972), 59f. (= v. Fritz, *Schriften zur griech. Logik,* Bd. 1: *Logik und Erkenntnistheorie,* Stuttgart 1978, 72f.). – Cf. also *infra,* n. 66.

2) D. J. Furley, *Two Studies in the Greek Atomists* (Princeton N. J., 1967), 72f., repr. in: A. P. D. Mourelatos, *The Pre-Socratics* (New York 1974), 362f., uses Simplicius' diagram and only studies Aristotle's text from *Phys.* Z 9, 240 a 9f onwards, omitting (I)–(III), for which see below, p. 4f. J. Barnes, *The Presocratic Philosophers,* Vol. I, *Thales to Zeno* (London 1979), 286ff., reconstructs the starting positions and movements of the bodies without trying to reconcile this with Aristotle's topographical indications.

I wish to start from a consideration concerned with a non-philosophical feature the four arguments against motion have in common: the fact that they are fun. They undoubtedly are very serious arguments, but they were also written in order to *épater le bourgeois*. The first argument proves that a runner will never get to the end of the stadium: once he has got half-way, he still has to get half-way the remaining half, half-way the remaining quarter, and so on[3]), *in infinitum*. The second proves that swift-footed Achilles will never catch up with the slowest thing on earth, because the distance in between, although constantly diminishing, forever remains proportionally the same. The third proves that a flying arrow, which occupies a place equal to its own size, is at rest, because it does not move at the place where it is, and not at the place where it is not either.

The first three arguments, then, are genuine and rather hilarious paradoxes. They reveal Zeno as a wit. To ask what is so funny about the fourth argument against motion therefore is a legitimate question. Yet I have hardly ever read an account of the fourth paradox which brought out the inevitable smile fetched by the others. Instead, one finds complicated discussions about infinite divisibility *versus* discrete or granulary structure, and endless shufflings and reshufflings of the runners on the course.

There are several reasons for this unfortunate situation, the most important of which, I believe, is that both ancient commentators (to judge from Simplicius' account) and modern scholars have failed to distinguish (or to distinguish sufficiently) between *Zeno's* paradox on the one hand and *Aristotle's* refutation on the other. Another reason is that Aristotle's text is plagued in parts with *variae lectiones* that seriously affect the meaning of the argument as a whole. Some of these readings enjoy the support of Simplicius, but this does not prove them right, for Simplicius points out one passage where Alexander of Aphrodisias followed a reading different from that accepted by

An amusing consequence – which, however, is not fatal to the interpretation proposed – is that Barnes' B's and C's move the wrong way, viz., his B's from right to left, his C's from left to right. – Cf. also *infra,* n. 42, n. 43, and text thereto. – The present paper had been finished before I was able to consult R. Ferber, *Zenons Paradoxien der Bewegung und die Struktur von Raum und Zeit* (Zetemata 76, München 1981); see further *infra,* n. 16, n. 17, n. 18, n. 29, n. 37, n. 40, n. 45, n. 59.

3) Or, alternatively: he has to get half-way before he gets to the end, and again half-way that half, and so on.

himself and which, as he believes, Alexander "found in some manuscripts" (*ἔν τισιν ἀντιγράφοις εὑρών*, *In Phys.* 1017, 19). Furthermore, as Simplicius likewise tells us (*In Phys.* 1019, 27–31), Alexander proposed to interpolate *Phys.* Z 9, 240 a 15–16 *ἴσον–φησιν* immediately after 240 a 11 *διεξεληλυθέναι*[4]). Alexander, then, found it difficult to understand the argument of the text as transmitted (which, at at least one other point, differed from Simplicius'). Simplicius' lengthy reconstruction of the fourth argument against motion and of Aristotle's critique thereof (*In Phys.* 1016, 7–1020, 6, printed – as far as 1019, 9 – by Lee as T 36) appears to have no other authority than his own, for he differs from Alexander, and the only other person cited (Eudemus, Fr. 106 Wehrli) is only adduced for points which do not affect the interpretation of the more difficult parts of *Phys.* Z 9, 239 b 33–240 a 17. Although scholars have dealt rather freely with Simplicius' commentary, using only those sections which fit their own views, it should be acknowledged that his reconstruction of the paradox, and especially his diagram of the stadium figuring three rows of runners, have been of crucial importance to the modern history of interpretation of Zeno's argument. I believe, however, that Simplicius (and perhaps Alexander as well) already made the fundamental mistake of failing to distinguish in the proper way between Zeno's paradox and Aristotle's refutation, although in Simplicius' case this is somewhat mitigated by the fact that he apparently noticed the joke of Zeno's argument (one doesn't know if Alexander did).

We are not bound, then, to follow Simplicius all, or even half the way, and need not even accept his guidance as to the choice to be made among the *variae lectiones*[5]). These different readings themselves, so it seems, reflect different ancient *interpretations* of Aristotle's exposition. In some manuscripts, *interpretamenta* may have got into the text (as at 240 a 6), or even have ousted other, more difficult readings (as at 240 a 11)[6]).

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4) For Alexander's modern followers, see *infra*, p. 21; and n. 59.

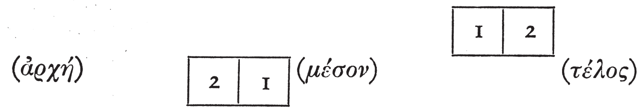
5) Cf. R. Mondolfo, *Problemi di pensiero antico* (Bologna 1935), 126f.

6) See further *infra*, p. 10f., 14f., where I argue that 240 a 6 *ἀπὸ τοῦ μέσου τῶν A* (instead of *ἀπὸ τοῦ μέσου*) and 240 a 11 *παρὰ πάντα τὰ A* (instead of *παρὰ πάντα τὰ B*) are to be rejected, although these readings are consistent with Simplicius' (and Alexander's, see *infra*, n. 60) interpretation – which presumably influenced part of the textual tradition. – Ferber, o. c., 14f., presents no arguments in favour of the *lectiones* which (wrongly, I believe) he prefers.

Zeno's paradox: the stadium

(I), Arist., *Phys.* Z 9, 239 b 33–240 a 1: "The fourth argument is the one about the (bodies) that move in the stadium in opposite directions, equal bodies past equal (bodies)[7]), the ones from the end of the stadium, the others from the (*sc.* its) middle, with equal velocity. This, he thinks, involves the conclusion that half the time is equal to (its) double".

The first problem to be tackled is that concerned with the topography of the stadium and the position of the running bodies, viz., the problem of the interpretation of *κινουμένων ... τῶν μὲν ἀπὸ τέλους τοῦ σταδίου, τῶν δ' ἀπὸ μέσου* (*sc.* *τοῦ σταδίου*). Lee[8]), following Simplicius[9]), argued that *τέλος* = the end (finish) of the race-course, and *μέσον* the middle of its length. He compares 240 a 5–6, *ἀρχόμενοι ἀπὸ τοῦ μέσου* (or, *pace* some manuscripts, *ἀπὸ τοῦ μέσου τῶν A*, the A's being a row of stationary bodies[10])), and quotes Gaye's explanation that the point of view is that of an imaginary beholder standing at the *ἀρχή* of the race-track[11]): both the row of bodies beginning at the middle-point of the race-course and that beginning at the end would stretch from those points (middle and end) towards the beholder at the starting-post:



7) Lee, *o.c.*, 55, translates "past each other", which is what must be meant. H. Wagner, *Aristoteles. Physikvorlesung* (Arist., *Werke in dtsch. Übers.* Bd. 11, Darmstadt 1972), 174, 10f., translates: "Es sollen zwei gleichgroße Fahrzeuggruppen aus entgegengesetzter Richtung *eine dritte Fahrzeuggruppe* [my italics] entlangfahren, die wiederum genau so lang ist wie jede von ihnen selbst". Wagner, therefore, detects a reference to the A's in *παρ' ἴσους*, but fails to explain why. His translation is also wrong in that it fails to bring out that the *individual* bodies are equal to one another (cf. *infra*, n. 29, and text thereto); the rows, of course, are too, but this only follows from the additional condition that the equal bodies in the rows are *ἴσοι τὸν ἀριθμόν*, as is explicitly, not superfluously, stated at 240 a 6 and 7–8.

8) *o.c.*, 85f.

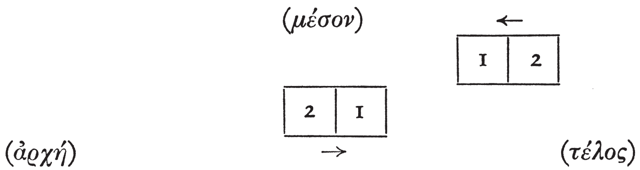
9) *In Phys.*, 1016, 27–1017, 5; 1017, 10–13.

10) *O.c.*, 85f., 90. Following Simplicius, *In Phys.*, 1016, 24ff., esp. 1017, 11 *ἀπὸ τοῦ μέσου τοῦ σταδίου, ἐν ᾧ καὶ τῶν A τὸ μέσον ἦν*, Lee argues that the middle of the stadium coincides with the mid-point of the row of stationary A's. See further *infra*, p. 12, and *supra*, n. 6.

11) Lee, *o.c.*, 89.

But this was refuted by Ross[12]), who pointed out that the bodies, according to (I), *move* from middle and end. Therefore, Ross argued, τὸ μέσον is the turning-point in a *double* course (*diaulos*)[13]), which would entail that τέλος is both finish and start. However, there is no evidence that μέσον was ever used in this sense, as Lee pointed out[14]) and Ross admitted[15]). We should, therefore, stick to μέσον as the middle point of the race-track, midway between start and finish. [The topography and terminology are the same as in the *first* argument against motion (*Phys.* Z 9, 239 b 11f.; 2, 233 a 21f. = *Vors.* 29 A 25), usually called the Dichotomy, which Aristotle himself once refers to as "the Stadium" (*Top.* Θ 8, 160 b 9 τὸ στάδιον). Here we have only one moving object, or runner, of whom Aristotle says that he has to reach τὸ ἥμισυ – i.e., must have run exactly half the distance – before he reaches τὸ τέλος, i.e., the finish. It is perfectly legitimate to infer that this lonely runner, in order to get to the τέλος after having run τὸ ἥμισυ, must start ἀπὸ μέσου.]

There is no problem in the stadium. The runners move at equal speed. The first group is said to move ἀπὸ τέλους, which means "away from the finish" or "from the direction of the finish", viz., towards the μέσον. On its way there it will meet the other group, moving ἀπὸ μέσου, "away from the middle", or from the direction of the middle, viz., towards the τέλος. The position of the moving groups as described in (I) is therefore *symmetrical*, not a-symmetrical. Consequently, we may combine Lee's description of the stadium (but not his positioning of the bodies) with Ross' positioning of the moving bodies (without accepting that μέσον = turning point), and express this by means of the following diagram[16]):



12) W.D.Ross, *Aristotle's Physics* (Oxford 1936, ²1955), 663; cf. also Ferber, *o.c.*, 16, 24.

13) *O.c.*, 663f. He was followed by J.H.Robinson, *An Introduction to Early Greek Philosophy* (Boston etc. 1966), 135f.

14) *O.c.*, 87; cf. Ferber, *o.c.*, 24.

15) *O.c.*, 663; cf. Ferber, *o.c.*, 24.

16) I am delighted to find the same diagram in Ferber, *o.c.*, 17 (who

The action to be analysed, in other words, occurs in only *half* the stadium, viz., in the part of the course between *μέσον* and *τέλος*.

Zeno's paradox: the joke

What is the reason for this complication arrangement? Why one row of runners *ἀπὸ μέσου*, another one *ἀπὸ τέλους*? These are obvious questions; they can be answered. One of Zeno's initial conditions is that all runners, being of equal size, move with equal velocity. His inference from what happens as these do pass one another, according to Aristotle, is that half the time is equal to its double. If one supposes the runners to have started (from *ἀρχή*) simultaneously and to have continued with equal velocity, the situation set out in the diagram could never have arisen. *Zeno's point, however, is precisely that it has.* Of two sets of runners running with equal velocity, the first row moves exactly twice as fast as the second, and this is why the first is now in a position *ἀπὸ τέλους*, the second only *ἀπὸ μέσου*. The first has run the distance s (s = stadium), the second the distance $\frac{1}{2}\ s$. Let us call t the time necessary to run the whole distance s. Because both groups have moved with *equal* velocity, the second group has run $\frac{1}{2}\ s$ in t, the time the first group needed to run the whole length of the stadium; so the runners in the first row have run $\frac{1}{2}\ s$ in $\frac{1}{2}\ t$. Therefore, $\frac{1}{2}\ t = t$. The position of the rows in the stadium at the beginning of Aristotle's transcript of Zeno's paradox reflects the fact that, in the world of Zeno's Stadium, half the time has been equal to its double all the time. Simplicius probably saw the idea[17]), but, if he did, he failed to see what it entails for the positioning of the runners.

The moment we enter the paradoxical world of Zeno's Stadium, then, the moving rows have passed the opposite ends

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however adds the A's, wrongly positioning them half-way the second half of the stadium; his diagram at p. 25 is confusing, for there A is the symbol of a moving body).

17) *In Phys.*, 1016, 10f.: "If motion exists, of two bodies of equal dimension and (moving) with equal velocity, the motion of the one will be double that of the other and not equal, in the same time. Which is absurd. Equally absurd is its corollary, viz., that the time, which is both the same and equal, is at once double and half". Barnes' reconstruction (which I do not follow for *philological* reasons), *o.c.*, 287f., takes cognizance of this absurdity. – Ferber, *o.c.*, 17, 25, argues that the C's do *not* move twice as fast as the B's and thus misses the essential feature of Zeno's paradox.

of the second half of the course. They will meet when the first row has got as far as ⅔ of this second half, and the second ⅓, for the first row moves twice as fast as the second:

(ἀρχή) (μέσον) × (τέλος)

Two rows or three?

All the diagrams illustrating the stadium I had seen before I saw Ferber's book invariably pictured three rows of runners: two moving, one stationary[18]). It was invariably assumed that Zeno spoke of *three* rows. But he did not, as is clear when one re-reads (I)[19]); I have read this text with a class of classical students, not future philosophers, several times now, and have asked each class, after we had translated (I) together: how many rows do you count? The answer has always been: *two* (Lee brings this out, in his translation[20]), but forgets about it almost immediately).

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18) To begin with Simplicius', *In Phys.*, 1017, as reconstructed by Diels. See further e.g. Diels-Kranz, *Vors.* I, 254; Lee, *o.c.*, 90; W.K.C. Guthrie, *A History of Greek Philosophy*, Vol. II, *The Presocratic Tradition from Parmenides to Democritus* (Cambridge 1965), 94; Furley, *o.c.*, 72 = 362; M.C. Stokes, *One and Many in Presocratic Philosophy* (Cambridge, Mass. 1972), 185; K. v. Fritz, *o.c.*, 60 (= 73). For a somewhat different one (of 3 × 4) see M. Untersteiner, *Zenone. Testimonianze e frammenti* (Firenze 1963), 152, 160f. cf. *infra*, n. 31; this he calls Alexander's, I do not know on what authority; perhaps he simply follows Diels-Kranz, *Vors.* I, 254, 21, who call Simplicius' diagram "Alexanders Figur" (why?). Untersteiner's figure does not fit the text (so, following Szabó, he has to argue (160) that one has to suppose "un fraintendimento del testo zenoniano da parte di Aristotele"). Barnes' diagrams, *o.c.*, 287f., descend from Simplicius', too, although he splendidly uses rows of *two* bodies. Bertrand Russell, in: W.C. Salmon (ed.), *Zeno's Paradoxes* (Indianapolis and New York 1970), 53, has three rows of *three* bodies (so also Salmon himself, *o.c.*, 11), but this, I would say, is ruled out by *Phys.* Z 9, 240 a 11–12, τὰ ἡμίση. Cf. Simpl., *In Phys.* 1016, 22f. ἄρτια μόνον, ὥστε ἔχειν ἥμισυ ἰσόογκα, and *infra*, n. 29. – I am delighted to note now that Ferber, *o.c.*, 24f., also argues that Zeno's paradox (according to (I)) is concerned with *two moving* rows only; however, in his reconstruction of the paradox, *ibid.*, 22f., the fact that these rows *pass along each other* plays no role whatever; cf. *infra*, n. 45, n. 29.

19) *Supra*, p. 4f.

20) *O.c.*, 55: "The first is ... about the two rows ... which move past each other ..., the one row ..., the other ...". Cf. also H. Boeder, *Grund und Gegenwart als Frageziel der frühen griechischen Philosophie* (The Hague

The third row, that of stationary runners, is *introduced by Aristotle* in order to *refute* Zeno. This should be clear from the text:

(II), *Phys.* Z 9, continued (240 a 1–4): "The fallacy lies in the assumption that a body of equal size takes an equal time in moving with equal velocity alongside another such (body) which is in motion and again another (such) one which is at rest. This is false".

The sequel, or at any rate the final part, of Aristotle's argument indeed operates with *three* rows (see *infra*, p. 20f.). Confusion has resulted from the attribution of all three rows *to Zeno*. Some scholars have thought Zeno's argument (assuming it involves three rows) silly[21]. Others believe that Aristotle misunderstood a profound or at least a cogent argument, which, occasionally appealing to a wealth of advanced mathematical subtleties, they endeavour to reconstruct on Zeno's behalf[22]. A good many vexing questions vanish, however, as soon as we make the proper distinction between Zeno's argument and Aristotle's criticism. It has to be admitted, of course, that Aristotle's exposition is not exactly lucid or straight-forward. In (I), he only gives us (*a*) the *initial conditions* prevailing in the Stadium and (*b*) Zeno's paradoxical *inference* about time, without telling us *how Zeno inferred (b) from (a)*. In (II), he immediately tells us what, in his view, is wrong with Zeno's argument, and does so even though we have not been permitted to follow this argument

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1962), 208f., esp. 209 n. 2, who speaks of two moving rows only, considers the stadium itself ("die Wegstrecke" as "eine ... Reihe von Raumeinheiten") as the third, and offers a conventional interpretation.

21) So already Eudemus, fr. 106 Wehrli, *ap.* Simpl., *In Phys.*, 1019, 32f.: *εὐηθέστατος ὢν* (*sc.* Zeno's *λόγος*), *ὥς φησιν ὁ Εὔδημος, διὰ τὸ προφανῆ τὸν παραλογισμὸν ἔχειν*. See further e.g. E. Zeller, *Die Philosophie der Griechen*, Vol. I.I (Leipzig [6]1919, repr. Darmstadt 1963), 760f.; N. Booth, *Zeno's Paradoxes*, JHS 1957, 194, and Guthrie's comment, *o.c.*, 94f. – If Zeno's argument really was silly, i.e., already included the stationary row which I believe was added by Aristotle, one fails to understand why people up to Aristotle's own time were unable to solve the problem (cf. *supra*, n. 1).

22) The Tannery tradition; cf. Guthrie, *o.c.*, 95f., and especially Stokes' critical discussion, *o.c.*, 184f. Stokes' own explanation of Zeno's argument, however, is far from simple, as is also Barnes', *o.c.*, 290f., who 'reconstructs' an original argument for Zeno which is not in Aristotle's text. A much simpler, and much more acceptable, interpretation is that given by Furley, *o.c.*, 74f. = 364f. (see further *infra*, n. 29, also for Ferber's proposal).

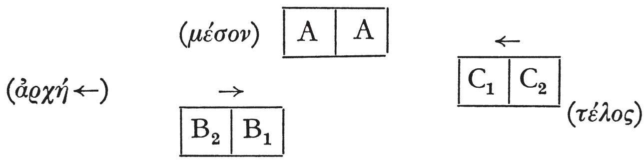
for ourselves. As we shall see, in (III) he does not just set the stage for Zeno's paradox, but immediately adds the stationary bodies, introduced in (II), to Zeno's moving ones taken over from (I). It is only in (IV), (V) and (VI) (see *infra*, p. 13f.) that he allows us to see in what way Zeno argued.

Aristotle, then, blends report with criticism, a procedure which we may deplore, but which, for him, is far from unusual[23]. In the present passage, however, report and comment have not been blended inextricably; or rather, Aristotle's construction has not destroyed the Zenonian level. We must try to excavate Zeno's original argument.

Aristotle's diagram

(III), *Phys.* Z 9, continued (240 a 4–8): "E.g., let AA be the stationary bodies of equal extension; BB those starting from the middle, equal to these [*sc.* the A's] in number and extension; and CC those (starting) from the end, equal to these [*sc.* the A's and the B's] in number and extension, and equal to the B's in velocity".

In (III), Aristotle assigns different symbols to the different rows of bodies. He first mentions the stationary ones, interpolated by himself. The moving bodies originally featuring in Zeno's paradox as of now are called BB and CC:



Aristotle's order is quite natural. Having first mentioned his stationary bodies – these, after all, are what his refutation turns on – he points out the two moving rows from left to right.

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23) H. Cherniss, *Aristotle's Criticism of Presocratic Philosophy* (Baltimore 1935, repr. New York, 1973), *passim*, and esp. 154–161, on Aristotle's treatment of the four arguments against motion (I cannot follow Cherniss all the way; note that *ibid.*, n. 74, he uses Simplicius' diagram of three rows, for which see *supra*, n. 18).

Aristotle's language, especially his use of letter-symbols and of the word ἔστωσαν, can not only be paralleled abundantly from his own works, but also from Euclid[24]).

In translating Aristotle's words in (III) as I have done, I have tried to keep the section of the text studied so far as simple as possible. Accordingly, I have assumed that ἀπὸ τοῦ μέσου at 240 a 6 means exactly the same thing as ἀπὸ μέσου at 239 b 35, and refers to the middle of the stadium. However, a number of manuscripts – and, apparently, Simplicius[25]) – read ἀπὸ τοῦ μέσου τῶν *A* at 240 a 6.

This gives a totally different sense: "let BB be the bodies beginning at the middle of the A's". Here "middle" is to be interpreted as the middle point of the line formed by the contiguous A's[26]). If this reading is adopted, we must suppose that Aristotle now adds an *extra* initial condition, viz., the exact determination of the position of the B's in relation to the A's. Naturally, some scholars who interpret the text in this way consistently translate 240 a 7 οἱ δ' ἐφ' ὧν τὰ ΓΓ ἀπὸ τοῦ ἐσχάτου (*sc.* ἀρχόμενοι) as "the C's begin at the end", i.e., of the A's again, the "end" being the end-point of the line formed by the contiguous A's[27]). Others, however, somewhat inconsistently I think, translate ἀπὸ τοῦ ἐσχάτου as "from the end of the course", and put the C's wherever it suits their argument[28]).

I find this unsatisfactory for several reasons. First, the addition of an extra condition, viz., the determination of the exact positions of the B's (and C's) relative to the A's, is superfluous. Secondly, it is not obvious why the obviously *discrete*[29]) equal

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24) Cf., e.g., *Elem.* I, prop. 3. – An interesting Aristotelian parallel is found in his discussion of Zeno's first argument against motion, *Phys.* Z 2, 232 a 34–b 11: the diagram is Aristotle's, not Zeno's. – Ferber, *o.c.*, 23, says ἔστωσαν and the use of letter symbols are typically Aristotelian, and that the latter are "bei den Eleaten nicht nachweisbar".

25) Simplicius does not quote this portion of the text; cf., however, his interpretation, 1017, 4. Lee, *o.c.*, 54, deletes τῶν *A*, which, he argues (*o.c.*, 90), is superfluous. Ross omits the words from his text of the *Phys.* Barnes, *o.c.*, 286, retains them; Ferber, *o.c.*, 17, rejects them.

26) Cf. *supra*, n. 10.

27) E.g., Barnes, *o.c.*, 287f., cf. esp. his fig. 1.

28) See, e.g., Lee, *o.c.*, 90.

29) This is an important issue in the *philosophical* interpretation of the Stadium. One has to grant Furley (*o.c.*, 74f. = 364f.) that Zeno's argument does not presuppose, *pace* Tannery and others (cf. *supra*, n. 22), the assumption of *indivisible minima*. Furley himself, however, argues – correctly, I believe – that place, motion, and time can be divided into "sections" which

bodies should be imagined as forming a *continuous*[30]) line. Thirdly, the most obvious translation of *ἀπὸ τοῦ μέσου τῶν A* is "from the middle *one* of the A's"; this, however, does not fit the context, for the bodies are *even* in number (cf. 240 a 11–12, *τὰ ἡμίση*, "half" the *bodies*[31]), and, apart from this, the translation "from the mid-most A" – the A's being uneven – would upset the required initial relative positions of the A's, B's, and C's. Fourthly, ms. support for *τῶν A* is relatively weak, and it is easier to understand these words as a marginal gloss[32]) scribbled by a person who desperately tried to figure out the puzzle than it is to ex-

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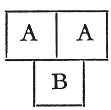
exhibit a one-to-one correspondence. These sections, I would say, are purely arbitrary (as big as a runner, or as an arrow). Zeno, however, must have meant that they are to be treated *as if* they could not be broken up or further divided (division would only produce new and smaller such "sections" with one-to-one correspondence). This is why Zeno (1) operates with a *plurality* of bodies and (2) says that *all* these bodies are equal to each other. If he had not wished to emphasize the arbitrary discreteness of place (time and motion), he could have used *two* bodies only, one moving from right to left, the other from left to right. Note that Aristotle, 240 a 11–12, says *παρὰ τὰ ἡμίση*: he thinks of discrete bodies, not of a continuous whole of bodies, for which *τὸ ἥμισυ* would have sufficed (note that a few mss. actually have *ἡμίσυ*). For Eudemus, see *infra*, n. 51. [Stokes, *o.c.*, 185 ff., and Barnes, *o.c.*, 291 ff., argue that the fourth argument against motion is not concerned with indivisible minima. Barnes, *o.c.*, 291, says "there is no reason why he [*sc.* Zeno] should himself have invented such a theory simply to knock it down". But did he not invent splendid assumptions about the infinitely many simply to knock them down?] – Ferber, *o.c.*, 26f., argues that Zeno was concerned with the idea that a part of a transfinite set is equal to the whole. Apart from the anachronism involved, however, this suggestion misses the point, for Zeno's inference that *half* the time is equal to the whole is both more narrow and more precise than the general statement that each subset of a transfinite set is equal to the whole set; Zeno proves that $1/2 = 1/1$ not by claiming that this is an instance of a property of transfinite sets, but in a totally different way. Ferber has missed this proof, because he refuses to attribute the greater part of (IV)-(V)-(VI), i.e., *Phys.* Z 9, 240 a 4–17, to Zeno. See further *supra*, n. 17, n. 18; *infra*, n. 37, n. 40, n. 45; and text thereto.

30) They are, of course, contiguous.

31) See *supra*, n. 18, *in fine*. It is, of course, true that one could suppose that $1/2$ A is involved, but this would involve that one B is opposite to two different $1/2$ A's, etc. (as in Mondolfo's diagram, see *supra*, n. 18):



32) So Lee, *o.c.*, 90.

plain how they could have gone missing. Finally, *ἀπὸ μέσου τῶν Α*, "from the middle point of the A's", spoils the simplicity of the passage as a whole, for at 239 b 35 *ἀπὸ μέσου* means "from the middle of the course". Simplicius, followed by many moderns, ingeniously attempted to bypass this difficulty by contending that the middle point of the A's is the same as the middle point of the course[33]. However, if the B's start at the middle point of the A's = the middle point of the course, they either (1) have all to be positioned in the part of the course between start and middle, viz., stretching backwards from the middle point of the course[34] although moving in the opposite direction, an arrangement I have argued against *supra*, p. 4f., or (2) they have all to be located in the other half of the race course, stretching away from the middle in the direction of their movement; but then the first B will already have passed the last A, for only half the A's (on the assumption that the mid-point of the A's = the mid-point of the course) are in this part of the course, and there are as many B's as there are A's – which makes nonsense of the B's as described subsequently. If, on the assumption that the mid-point of the A's = the mid-point of the course, the B's cannot be placed, it is superfluous to attempt to locate the C's.

I therefore assume that 240 a 5–6, *οἱ ... ἀρχόμενοι ἀπὸ τοῦ μέσου*, refers to 239 a 33f., *κινουμένων ... τῶν δ' ἀπὸ μέσου* (*sc.* *τοῦ σταδίου*), and that 240 a 7, *οἱ ... ἀπὸ τοῦ ἐσχάτου* (*sc.* *ἀρχόμενοι*) refers to ibid., *κινουμένων τῶν μὲν ... ἀπὸ τέλους τοῦ σταδίου*. This gives perfect balance: in the diagram, the stationary row is contrasted to two moving rows, one starting from the left, the other from the right. The word *ἔσχατον*, here, is synonymous with *τέλος*.

On this assumption, it also becomes possible to explain why, at 239 a 33f., Zeno's paradox lists the row coming from the finish [the C's] *first*, and that coming from the middle [the B's] *second*, whereas Aristotle, setting out his diagram, reverses this order[35]. *The original order (in I) is essential to the paradox*, because it suggests that the C's already *have* moved twice as fast as the B's. For the diagram, this is irrelevant. Having first listed his own supernumerary row [the A's], Aristotle continues by simply enumerating the two other rows from left to right.

33) Cf. *supra*, n. 10, n. 26, and text thereto.

34) See the diagram in Simplicius, Lee, Guthrie, Furley, Stokes, von Fritz (*supra*, n. 18).

35) This problem does not seem to have bothered commentators.

What follows

According to Aristotle, several conclusions follow. We should first study the first of these, taking care, when translating, to preserve Aristotle's word-order:

(IV), *Phys.* Z 9, continued (240 a 9–10): "It then follows that the first B is at the last at the same time as the first C (is at the last), as they move past one another".

"At the last", ἐπὶ τῷ ἐσχάτῳ. This, at first, is difficult. We should rule out the meaning "finish" or end-point of the stadium, for only the B's move there. So the rather popular assumption that ἐπὶ τῷ ἐσχάτῳ refers to the ἔσχατον of a row or rows must be correct. What is more difficult is to establish the exact reference of this "last": is it the last point of a row of bodies to be thought of as forming a continuous line, or the last member of a set of runners? Since we have been able to follow the argument up to now without equating a row of discrete bodies with a continuous line, I submit that we plump for the second alternative, although this entails that ἔσχατον at 240 a 9 has another meaning (last *member*) than at 240 a 7 (last *point*). But this is not fatal. Because Aristotle says "the first of the B's is at the last ...", the inference that a noun, or rather a variable representing a noun, is to be supplied with ἐσχάτῳ, "last", smoothly follows from the fact that such a variable (C) indeed represents the noun to be supplied with πρῶτον. *The word* ἔσχατον *here is the opposite of* πρῶτον. It also helps to imagine Aristotle pointing out things in his diagram; the first B, *moving* from left to right on its way towards the finish (τέλος or ἔσχατον) of the stadium, at a certain moment arrives at the last body it has to pass, which, considered from the view-point of the first of the B's, is the last or hindmost C. For the B's, the last of the C's lies in the direction of the end of the stadium. We should, accordingly, think of bodies as being alongside or "at"[36]) other bodies, not of extended bodies being "at" a point.

It should be further noted that Aristotle, in the sentence at issue, speaks of *moving bodies* only, viz., of B's and C's moving alongside or past *each other* (παρ' ἄλληλα). *The A's do not enter into the picture at all.*

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36) For the "static vocabulary" used – at least in parts of the argument – by Aristotle see Furley, *o. c.*, 74. It should be noticed, however, that in the section as a whole, 'static' and 'kinetic' terms tend to be equivalent, as befits the paradox.

In the first part of this sentence, then, "the first B is at the last...", Aristotle, so to speak, moves along with the B's from left to right, until the first B is at the last C. He now makes a mental turn and follows the course of the C's from right to left. These, too, travel, and the last body at which the first C arrives is the last or hindmost B. So, the first B arrives at the last (C) simultaneously with the first C (arriving at the last B). This state of affairs is made explicit by the Oxford translation, and Lee[37].)

This first inference is not at all absurd or paradoxical, but states what happens, according to common sense, when two equal rows of equal bodies, going in opposite directions with equal velocity, move past one another.

What follows next

Aristotle's next inference presents special difficulties, because for the first part of the sentence (240 a 10 *συμβαίνει* – 12 *ἡμίση*) the manuscripts are divided: according to some of them, Aristotle here mentions the A's, according to the others – about the same as those omitting *τῶν A* at 240 a 6[38]) – he does not. The different translations that are possible, depending on the *varia lectio* chosen, are (1): "It further follows that the C has passed *all the B's*, and the B half"; and (2): "... that the C has passed all *the A's*, and the B half". Now (2) entails that the "half" passed by the B are A's, too[39]). Often, however, those who argue or believe that (1) is to be preferred assume that the "half" passed by the B must be A's[40]) [I shall call this (1 a)]; Simplicius already thought so, and in his paraphrase he even wrote *παρὰ τὰ ἡμίση A* (*In Phys.*, 1018, 1), whereas others conjecture *παρὰ ‹A› τὰ ἡμίση* or *παρὰ τὰ ‹A› ἡμίση*[41]), a slight but inevitable change in Aristotle's text.

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37) "the first B reaches the last C at the same moment [at the same time, Lee] as the first C reaches the last B". Ferber, *o.c.*, 17f., believes that the first C is at A1 at the same time as the first B is at A2, *quod non*.

38) Cf. *supra*, p. 10f.

39) So Barnes, *o.c.*, 286, 287 ("the first *C* has passed all the *A*s and the first *B* has passed half the *A*s").

40) So Ferber, *o.c.*, 14, 18.

41) So Diels-Kranz, *Vors.*, I p. 254, 13; G.S. Kirk–J.E. Raven, *The Presocratic Philosophers* (Cambridge 1957, [4]1963), 295. Lee, *o.c.*, 56, who reads *τὰ Γ* and *παρὰ τὰ ἡμίση*, translates the latter with "half that number of bodies ‹viz., two *A*s›", i.e., adds to the translation what others add to the

In themselves, both (1a) "the C has passed all the B's, and the B half ‹the A's›" and (2) "the C has passed all the A's, and the B half (sc. the A's)", can be explained quite plausibly[42]). My chief objection to both these interpretations, however, is that what results is a form of the paradox which contains its own refutation: the reference to the A's both times shores up *Aristotle's* criticism, and Zeno's paradox itself is lost[43]). Generally, those who think the A's have to be involved here have been obliged to reconstruct Zeno's original argument *ex hypothesi*. A reading of the treatment of the fourth argument which would allow us to recover Zeno's original argument from Aristotle's text would therefore seem much to be preferred. I shall argue in favour of such a reading by plumping for (1), which is consistent with the preference for the shorter version of the text (omitting *τῶν A*) at 240 a 6 for which I have argued above[44]). This entails that I opt for the same (or approximately the same) textual tradition both times. I therefore translate as follows:

(V), *Phys.* Z 9, continued (240 a 10–13): "It also follows that the C has passed all the B's, and the B half, so that the time is half. For each of these two is alongside each individual other for an equal time".

"The C" must be the *first* C (C1), mentioned in the previous sentence (IV). "The B" therefore has to be the *first* B (B1). It is stated that C1 has passed all the B's. It is also stated that B1 has passed "half" – no variable added. *Since only B's and C's are involved, the only possibility is that B1 has passed half the C's.* "Each of these two" (ἑκάτερον) therefore refers to C1 and B1. "Each individual other" (ἕκαστον) refers to each individual body in each row: to B1, B2, C1, C2.

The statement that C1 has passed *all* the B's when B1 has passed *half* the C's (all B's and C's being equal in size and moving

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text [N.B.: note that Lee operates with rows of four bodies]. For Ferber's view see previous n.; note that he does not discuss the problem of the *variae lectiones*.

42) For (1a), see, e.g., Furley, *o.c.*, 72f. = 362f., to be commended for its simple soberness; for (2), see Barnes, *o.c.*, 286ff., to be commended for its brilliance.

43) Note that Furley, *loc. cit.*, does not take *the whole* of Z 9, 239 a 33–b 18 into account, but only starts at 240 a 9, and that Barnes, *o.c.*, 290f., has to reinvent a paradox on Zeno's behalf (cf. *supra*, n. 2). – Ferber, *o.c.*, *loc. cit.*, has failed to acknowledge the consequences of a reference to the A's at this point.

44) *Supra*, p. 10f.

with equal velocity, and there being as many B's as there are C's) constitutes *a paradox*. Immediately before, in (IV), Aristotle had communicated the common-sense inference that C1 will be at the last B at the same time as B1 will be at the last C. At present, he infers that B1 will only have got past *half*, not all, the C's, when C1 has moved past *all* the B's.

The B's, moving with the same velocity as the C's, therefore only get half as far (or travel twice as slowly) as the C's. I find this paradox thoroughly satisfactory, for I have argued above (on grounds independent of the interpretation of the present passage)[45] that Zeno's joke actually was that the C's move twice as fast as the B's. The paradox inferred in (V) has precisely the same point as the original arrangement of the rows in the stadium at the outset of the fourth argument against motion (see I)[46]. The original setting therefore is a blow-up of the paradoxical situation in (V), or, conversely: the situation in (V) is a microscopic Stadium. How is this paradoxical result possible? Aristotle tells us – and be it noted that this part of the sentence is not subject to such doubts as accompany the presence of *variae lectiones* in its previous part – that *each first body*, i.e., both C1 and B1, *is alongside each individual body of the opposite row for an equal amount of time*. This important statement does not exactly tally with anything said by Aristotle previously, for we are talking now of *moving* bodies only. To be sure, at 240 a 1–4[47]) Aristotle had said that Zeno's *fallacy* was that he assumed that a body moving at equal speed past an equal body that is *at rest* and another equal body that is *moving* (in the opposite direction) needs an equal time to pass both. But I have argued above[48]) that the *stationary* body was *added* by Aristotle, i.e., that Zeno spoke of moving bodies only. If I am right about the portion of the text at issue now, again only moving bodies are involved, viz., C's and B's. Consequently, Aristotle, by telling us that each *first* body of a moving row has to be alongside *each individual* body (*ἕκαστον*) of the row moving at equal speed in the opposite direction for the same amount of time, all bodies, as we remem-

45) See *supra*, p. 6f. – Ferber, *o.c.*, 22, argues that the whole of (IV)-(V)-(VI), i.e. 240 a 4–17, belongs to Aristotle, not Zeno (cf. *supra*, n. 16, n. 18, n. 29, n. 37), and believes that Aristotle most probably "das Stadium gründlich mißverstanden hat".

46) *Supra*, p. 5.

47) Cf. *supra*, p. 8.

48) *Supra*, p. 7f.

ber, being equal in size, must be giving us the premiss *used by Zeno* to infer that "the time is half" – a premiss we found lacking[49]) in Aristotle's transcript of the fourth argument of motion at the beginning (I) of the section as a whole.

This recovered premiss, taken together with the initial conditions stipulated by Zeno in (I), entails that B1 has to be alongside (or to pass) C1 in time *t*, or that B1, in *t*, passes C1[50]). C1, however, is also moving, at the same speed as B1 (and as the other equal bodies concerned), but in the opposite direction; therefore, C1, in *t*, gets as far as the last of the B's. Consequently, C1 has passed all the B's in *t*, whereas B1, in *t*, has passed only one C, i.e., half the C's. Since *t* is the time which, according to the recovered premiss, a moving body needs to pass *one and only one* other body moving in the opposite direction, and C1 has passed *two* such bodies in *t*, C1 only needs ½ *t* to pass one body: its "time is half". This again is Zeno's paradox.

Excursus on the second inference

This paradox is feasible only on the assumption that motion consists of a series of *moves*, as of pieces on a chess- or draught-board: the "bodies" can be compared to the πεσσοὶ the Greeks used to play with. A further assumption is that each "body" is coextensive with the place or "square" it occupies[51]). A move can only be from one such place into another. The idea that a body is coextensive with its τόπος is essential to Zeno's third argument against motion, viz., that of the stationary flying arrow[52]). There is no anachronism involved in the suggestion that it is also operative in the fourth argument[53]).

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49) See *supra*, p. 8.

50) I assume that the 'static' vocabulary is equivalent to the 'kinetic'; cf. *supra*, n. 36.

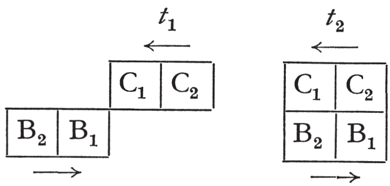
51) This does not entail that the bodies etc. are indivisible minima; see *supra*, n. 29. – The game can also be played by people sitting on opposite rows of chairs and moving up one chair in their own row. – Note that Eudemus *ap*. Simpl., *loc. cit.*, = Fr. 106 Wehrli, suggests that the ὄγκοι are *cubes*, which, although no more than a suggestion, may entail that he thought of *discrete* bodies.

52) *Phys.* Z 9, 239 b 5–9, 31–33 = *Vors.* 29 A27. See G. Vlastos, *A Note on Zeno's Arrow*, in: R. E. Allen – D. J. Furley (ed.), *Studies in Presocratic Philosophy*, Vol. II, *Eleatics and Pluralists* (London 1975), 184ff.; K. v. Fritz, *o.c.*, 63 (= 76). For τόπος see also Parm., *Vors.* 28 B8, 41, quoted *infra*, p. 24.

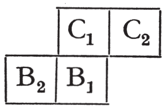
53) See further *infra*, p. 22. – The idea will have been inspired by

Body, motion, space, and time, therefore, in the world of Zeno's Stadium, are treated as being *discrete*, not continuous. The bodies represent spatial units. Motion comes in jumps, each of which corresponds to one such spatial unit: a body, according to the recovered premiss, moves one unit of space (one "square") equal to itself. Time ticks away in corresponding units: the unit of time is the time needed to get from one spatial unit into the next.

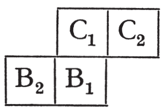
Zeno, of course, could also have argued that there is *no* time *t* during which C1 and B1 are alongside one another. When both move past each other at equal speed, covering the minimal distance (one body) during *t*, the following sequence occurs:



There is *no* time between t_1 and t_2, so there is no time either at which the following situation occurs:



But this is against common sense: surely, B1 and C1 must be alongside each other before C1 can be alongside B2 (and B1 alongside C2)! Common sense demands that



Parmenides, see *Vors.*, 28 B 8, 29–30 *ταὐτόν τ' ἐν ταὐτῷ τε μένον καθ' ἑαυτό τε κεῖται χοὔτως ἔμπεδον αὖθι μένει*. Parmenides' "Being" is – *mutatis mutandis* – as motionless as Zeno's arrow is. Note that Zeno argued that the arrow neither moves where it *is* nor where it *is* not.

occurs in *t*, and the recovered premiss not only allows this, but positively demands it. But C1 is moving as well, and at equal speed. Common sense, therefore, demands that it advances one spatial unit, viz., one body, and so posits it alongside B2, which, following B1, has, of course, moved too. Consequently, C1 is alongside B1 and B2 simultaneously, which is patently absurd: the situation presupposed by Zeno's paradox cannot be represented in a diagram[54])! Zeno has proved his point, viz., that motion cannot occur.

We are now in a position to appreciate the shrewdness – and novelty: for the puzzle had not been solved before Aristotle – of Aristotle's remark that Zeno's argument is *fallacious* in that he equates motion alongside a moving body with motion alongside a body that is at rest (see VI). For Aristotle, motion (like space and time) is not discrete, but continuous – not an actual, but a potential infinite. He has set this out at required length in the immediately preceding pages, arguing against Zeno's other paradoxes of motion. As soon as it is assumed that motion etc. is continuous, Aristotle's point follows, for Zeno's argument entails that C1 is *arrested*, or frozen, for the time B1 needs to come alongside it (just as, *mutatis mutandis*, the arrow's motion is frozen, *pace* the third paradox), whereas, simultaneously, it is allowed to *move* in order to arrive alongside the last B. It is possible, for Aristotle, to argue that Zeno simultaneously treats C1 as a moving and as a stationary body (an A, to use the terminology of Aristotle's diagram), which offends against the *principium contradictionis*. Aristotle's own row of stationary bodies added to Zeno's moving ones merely makes this point explicit – it was not absolutely necessary to introduce the A's, but their addition to Zeno's rows certainly clarifies Aristotle's argument[55]).

What follows third

(VI), *Phys.* Z 9, continued (240 a 13–17): "Simultaneously, however, it follows that the first B has passed all the C's, for the first B and the first C will be at the opposite last (bodies) simul-

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54) Cf. Barnes, *o.c.*, 288f., who similarly constructs a situation which cannot occur, although he is able to represent it in two diagrams – but he does not attempt to work out the intermediate stage between his figs. 1 and 2.

55) Cf. *infra*, p. 21f.

taneously, and it [*sc.* the first C] gets alongside each individual B for the same time as alongside each individual A, *pace* Zeno, because both [*sc.* the C's and the B's] get alongside the A's for the same amount of time".

To begin with, it should be noted that the sentence "the first B has passed *all* the C's" is the exact opposite of the sentence "the first B has passed *half* [*sc.* the C's]", to be found in (V)[56]). This symmetry confirms our interpretation that, in (V), the "half" passed by B1 are indeed half the C's. Next, the conclusion that B1 (as in VI) has passed all the C's is inferred, explicitly, from the premiss that C1 and B1 are simultaneously alongside the last members of the opposite row – a premiss proved in (IV)[57]).

Consequently, (IV), (V), and the first part of (VI), taken together, entail the further paradox (which I have already discussed, *ex hypothesi*, for C1 in the previous paragraph[58])) that B1 is simultaneously alongside C1 and the last C (C2). This is absurd. [For B1, which is able to get as far as C1 and as C2 in the same time, half the time is again equal to its double. Aristotle does not state this explicitly. He is not thinking of a body moving twice as fast as itself, but of one set of bodies, C's, as moving twice as fast as another set, the B's. I.e., he is still thinking in the terms of Zeno's original joke and not of continuous lines].

It is only now, I believe, namely in the second part of (VI), that Aristotle comes out with his refutation of Zeno's paradox, by adding, *for the first time*, an explicit reference to the A's.

Aristotle's refutation of Zeno's fourth argument

Aristotle says (in VI): the first C, a moving body, "gets alongside each individual [moving] B for an equal time as alongside each individual [stationary] A, as he [*sc.* Zeno] says, because both [*sc.* the B's and the C's] get alongside the A's for an equal time".

The first part of this sentence can be formulated more explicitly. C1 is alongside each B and each A for an equal stretch

56) Cf. *supra*, p. 15.
57) Cf. *supra*, p. 13f.
58) *Supra*, p. 19f.

of time. Hence also B1 is alongside each C and each A for this equal stretch of time. I shall call the time C1 is opposite each of the A's C*t*A, the time it is opposite each of the B's C*t*B, and similarly speak of B*t*A and B*t*C.

This part of the argument has puzzled ancient commentators and modern scholars: Alexander of Aphrodisias[59]), followed by e.g. Lee and Ross, wanted to transpose 240 a 15–16 *ἴσον χρόνον παρ' ἕκαστον γιγνόμενον τῶν B ὅσον περ τῶν A, ὥς φησιν* to 210 a 11, after *διεξεληλυθέναι* (with a bonus for their interpretation: for the *ἡμίση* of 240 a 12 would then be A's[60]).

Alexander and others, however, have not seen that Aristotle, not without humour, applies the axiom *τὰ τῷ αὐτῷ ἴσα καὶ ἀλλήλοις ἐστὶν ἴσα*[61]). Aristotle says that Zeno says that C*t*B = C*t*A *because* B*t*A = C*t*A. Because C1 and B2, passing each other as well as one of the A's, are alongside one of the A's during *t*, they are also alongside one another during *t*:

C
A
B

Aristotle is now in a position to end with his *quod non*:

(VI), *Phys.* Z 9, continued (240 a 17–18): "This is the argument, and it is conclusive in virtue of the false assumption stated previously", viz., in (II)[62]): Zeno's unit of reference is moving and stationary at the same time, which, of course, is against the *principium contradictionis*.

Aristotle's concluding remark refutes *both*[63]) foregoing

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59) *Ap.* Simpl., *In Phys.*, 1019, 27–32. – Ferber, *o.c.*, 14 (cf. *supra*, n. 45) athetises *ἴσον χρόνον – φησιν*, without giving a reason why. I assume he believes this sentence interferes with the suggestion that Zeno spoke of two rows only (cf. *supra*, n. 18, *in fine*).

60) Cf. *supra*, p. 3 f., and n. 6. Suggestions such as Alexander's may be responsible for the infiltration of the A's which I suppose to have occurred, i.e., the manuscripts which have the A's (see *supra*, n. 6) may represent a tradition which adapted Aristotle's text to Alexander's interpretation.

61) Furley, *o.c.*, 74, and Barnes, *o.c.*, 298, have seen Aristotle's point.

62) See *supra*, p. 8.

63) Note that Simplicius, *loc. cit.*, *supra* n. 17, states that there are *two* absurdities in Zeno's argument (these are not wholly identical with the two absurdities pointed out in the present paper).

paradoxes, i.e., both the paradox that C1 has passed all the B's when B1 has passed only half the C's, and its paradoxical corollary that B1, when it has passed only half the C's, i.e., is alongside C1, is simultaneously alongside C2.

It could be objected that Aristotle's ὥς φησιν [*sc.* Zeno], 240 a 16, indicates that he is actually quoting, or at least paraphrasing *what Zeno said*. In that case, Zeno would have dealt with *three* rows after all, and the argument of the present paper would have to be abandoned. But Aristotle often tends to attribute to his opponents the views or mistakes he imputes to them. So also here: the tacit – and fatal – presupposition (τῷ ... ἀξιοῦν[64]), 240 a 2–3) 'discovered' by Aristotle by now has become what Zeno 'said'.

This can be paralleled from Aristotle's treatment of Zeno's third argument against motion, concerned with the stationary flying arrow:

Phys. Z 9, 239 b 5–6 = *Vors.* 29 A27: "Zeno's argument is fallacious [παραλογίζεται, cf. 240 a 2 παραλογισμός]. For if, *he says* [φησιν, cf. 240 a 16, φησιν], everything is always at rest when it is at a place equal to itself, and the moving object is always (*sc.* occupying such a place) in the 'now', then the moving arrow is motionless". As G. Vlastos has shown, the "now" is an Aristotelian technical term. "Its presence here is explicable as an Aristotelian plant: by sticking it into his account of the puzzle, Aristotle makes it all the easier for his readers to feel the appositeness of his refutation, which centres in the claim that 'it [Zeno's argument] assumes that time is composed of "nows"': if this were not granted, the argument would not be valid"[65]).

One of the things for which I have argued in the present paper is that the A's of the fourth argument against motion are "an Aristotelian plant" too. I am convinced that he put them in on the same didactic grounds as induced him to interpolate the "nows" in the third argument. The third argument, inclusive of the "nows", is cited with a φησιν. Accordingly, the φησιν at 240 a 16 does not prove that Zeno spoke of *three* rows. Yet, if I have argued correctly, Aristotle is more careful in his exposition of the fourth than in his transcript of the third argument. Although he again states quite clearly, and almost immediately, that Zeno's argument is fallacious and why it is so, and although

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64) Cf. 239 b 26, τὸ ... ἀξιοῦν.
65) Vlastos, *o.c.*, 187.

he brings the A's into the Stadium rather early, he only *uses* these A's near the end of his exposition, where he winds up by showing why – in his view – Zeno was wrong, and how wrong – in his view – he really was. *In between (240 a 9–15) he had set out Zeno's argument quite correctly.*

Zeno's paradox and Aristotle's refutation

Although it has been argued recently that Zeno's four arguments against motion do not necessarily form a reasoned sequence[66]), I believe that there are certain indications that they do so. Aristotle himself points out that the first and the second paradoxes belong together or, rather, are really the same argument (*Phys.* Z 9, 239 b 18–9, ἔστιν δὲ καὶ οὗτος ὁ αὐτὸς λόγος τῷ διχοτομεῖν). The first two paradoxes are about place, motion, and time as being infinitely divisible. I have argued above[67]) that the Stadium, i.e., Zeno's original Stadium as reported by Aristotle, is about *discrete* units (ὄγκοι), occupying places equal to themselves, and needing an equal time t to move from one place into the next. The third paradox was about a moving body that was simultaneously stationary: to those who said "eppur si muove" or its equivalent in Greek, Zeno, I suppose, would have spelled out his fourth argument, which likewise shows that a moving body (C1) is both moving and stationary[68]); the idea behind the third paradox can also be found in the fourth. In all his arguments, Zeno starts by granting his opponents that motion exists, i.e., he starts by arguing, as the true "father of dialectic"[69]), from the premiss proposed by his opponents, and proves that it is indefensible, because it entails mutually contradictory conclusions. The third and fourth paradoxes against

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66) This used to be assumed, cf., e.g., G.E.L. Owen, *Zeno and the Mathematicians*, in Salmon (ed.), *o.c.*, [139ff.], esp. 150. Against Owen see Stokes, *o.c.*, 189f., and Barnes, *o.c.*, 285, 291. It used to be said that the first two paradoxes are about place etc. as infinitely divisible, the last two about place etc. as consisting of indivisible quanta or minima. For my own view see *supra*, n. 29. – Cherniss, *o.c.*, 155, states that Aristotle gives the order of the arguments as original; an important observation.

67) See *supra*, p. 10f., and n. 29, n. 18.

68) Cf. *supra*, p. 19.

69) Aristotle, fr. 65 Rose *ap.* Diog. Laert. VIII 57 = *Vors.* 29 A10. Cf. Lee, *o.c.*, 7f., 113f. See also Isocrates, *Helen.*, 3: "Zeno, who tried to prove successively that the same things are possible and impossible".

motion avail themselves of the subsidiary premiss that motion can be chopped up into a finite amount of little moves, the first and second that motion consists of infinitely many moves. It possibly goes too far to suspect that Zeno's arguments, from one to four, unfold a grand design. There is nothing I can see, however, against the assumption that he explored all the possibilities.

It is interesting to note that Aristotle's refutation of the fourth paradox, for all its cleverness, suffers from *petitio principii*. I have already recalled[70]) that his argument *contra* Zeno is grounded in his own conviction that place, motion, and time are potentially infinite continua. However, his special point against the fourth paradox, viz., that Zeno's mistake (or, as Eudemus said, "stupid mistake"[71]) is that he assumes that a moving body of size *z* needs an equal time to pass both another body of the same size *z* *moving* in the opposite direction and another *stationary* body of size *z*, introduces a *distinction between motion and rest* which he apparently takes for granted: motion and rest are contradictories. Zeno's point, on the other hand, really is that, in order to speak cognitively about motion, you have to distinguish between motion and rest, but that – as his arguments show – you cannot do so. The moving arrow freezes, the first of the C's is at rest while moving. Motion, as distinguished from rest, is, for Zeno, the great *probandum*. One may assume that he accepted Parmenides' proof that "what is" is motionless (*Vors.* 28 B8, 26–31). Parmenides, of course, had already argued that the assumption that *τόπον ἀλλάσσειν* is possible is a mere human error (ibid., 39–41 *ὅσσα βροτοὶ κατέθεντο πεποιθότες εἶναι ἀληθῆ, ... (sc.) τόπον ἀλλάσσειν* etc.). Zeno's opponents, in other words, had already been identified by Parmenides.

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70) *Supra*, p. 19.
71) Cf. *supra*, n. 21.