Introducing Data-Based Modeling with Functions in a Student Laboratory: Design and Implementation for Enhancing Statistical Literacy among Upper Secondary School Students

MATTHIAS MOHR, LMU MÜNCHEN; LUZIA HOFER, LMU MÜNCHEN & STEFAN UFER, LMU MÜNCHEN

Abstract: *This paper presents the development and implementation of a process framework for data-based modeling of functional relationships between two numeric variables, designed as an introductory experience for upper secondary school students. We describe the design process and the current concept of a student laboratory, that integrates this process model into a learning setting, following specific content and methodological design principles. Our approach aims to provide students with initial opportunities to engage with real data and mathematical modeling in meaningful contexts, thereby advancing their statistical literacy.*

1. Introduction

The need for statistical literacy is greater than ever. Information about (global) topics such as the Covid-19 pandemic, global warming, extreme weather conditions, or economic growth are presented in the form of visualizations, scientific studies, or data sets and are widely spread via a variety of channels—websites, social media, television, radio, and so on. Given the ease and minimal regulation of accessing information today, young people need to develop basic statistical literacy as early as possible to be able to understand and process such information. However, this requires data science education to be firmly anchored in the curricula at the school level (e.g., Biehler & Schulte, 2018).

Data-based modeling represents a possible approach to engage students with the principles of statistical literacy (Engel & Kuntze, 2011). In data-based modeling with functions, a relation between two observed variables is described with a suitable mathematical function based on real data (Eichler & Vogel, 2013; Engel, 2018), often to answer a concrete question. This process comprises generating one or more competing functional models that reflect assumptions about the relation between the variables and fitting them to a given dataset. The resulting models can be compared and validated to select an optimal model, which is interpreted to answer a question, for example about the context from which the data originates. Corresponding concepts and practices have received increased attention in this field as important learning goals of mathematics teaching (e.g., Fleischer et al., 2022; Podworny et al., 2022; Vogel & Eichler, 2014). However, little is known about the processes and determinants for the development of skills in data-based modeling. Furthermore, there are few concrete approaches to implement data-based modeling in the school context (Biehler et al., 2011; Pfannkuch et al., 2018).

Starting from Gal's (2002) model of statistical literacy, we describe theoretical knowledge and practical skills that can be addressed in the context of data-based modeling. We describe the design and current concept of a student laboratory that serves as a concrete approach to implement data-based modeling in a school-like setting. In this sense, a student laboratory can represent a controlled, extracurricular learning setting that can be used to pilot educational approaches. It provides students with the opportunity to collaborate in small groups under guidance by pre-service teachers, and to expand their knowledge and skills in specific subject areas through hands-on experimentation, independent problem-solving, and exploration (Ralle, 2020).

This contribution pursues the following goals: (1) Presenting the development of a process framework for data-based modeling, intended for use in a student laboratory and designed as an initial introduction to data-based modeling. (2) Describing the design process and the current concept of a student laboratory for upper secondary school students, which implements this process model into a learning setting. (3) Discussing how this approach can address aspects of statistical literacy by providing students with opportunities to engage with real data and mathematical modeling.

2. Data-based modeling with functions in the field of statistical literacy

With the uprise of competence as a central goal of mathematical school instruction (Niss, 2003; Weinert, 2001), and driven by the focus of large scale assessments such as PISA on the concept of (mathematical) literacy, the acquisition of applicable knowledge has been re-emphasized in mathematics education research (e.g., Vogel, 2014). This places a strong emphasis on connecting mathematical learning with real-world phenomena. This aims, on the one hand, to empower students to use mathematical concepts for active exploration of their environment and, on the other hand, to develop resilient mental models for mathematical terms (Freudenthal, 1983). Niss et al. (2007) emphasize primarily the relevance of mathematics to reality and its modeling nature.

2.1 Characterizing statistical literacy

Following Gal's (2002) definition, statistical literacy comprises „the motivation and ability to access, understand, interpret, critically evaluate, and if relevant express opinions, regarding statistical messages, data-related arguments, or issues involving uncertainty and risk". This definition describes several aspects of a complex construct that goes much further than just reading and evaluating data and diagrams. In his model, Gal (2002) describes different building blocks of statistical literacy, which are roughly divided into the areas of knowledge and dispositional elements (Fig. 1). The knowledge elements include literacy skills, statistical knowledge, mathematical knowledge, context knowledge, and critical skills (questioning sources and the possibility for biased reporting) and the interaction of which together stands for the ability to comprehend statistical information, interpret, and critically evaluate. The dispositional elements contain beliefs and attitudes towards statistics as well as critical stance. Critical stance refers to a selective attitude regarding the uncertainty of statistical information and to question it in a meaningful way (Where did the data come from? Is the sample representative? How reliable or accurate were the measures? Are there plausible alternative interpretations?).

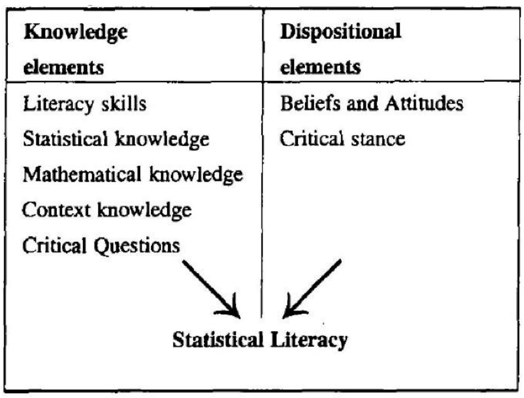

Fig. 1: Gal's model of statistical literacy

Gal et al. (2022) extend their model towards civic statistics, which describes the knowledge and skills needed by individuals to critically understand statistical information and that underlying societal and economic problems. More specifically, they subdivide these requirements into engagement & actions, knowledge, and enabling processes. Under engagement & actions they subsume learners understanding why data is needed and recognize the meaning and social implications for society and policy. This also includes the need for critical evaluation and reflection of data sets, statistical models, and results. Equally, they also list motivational dispositions here, including a complex web of beliefs and attitudes for engaging with evidences. The knowledge area comprises facets of knowledge of statistical concepts and methods, qualitative and quantitative methods, to interpret and analyze models and their development processes, but also contextual knowledge about the phenomena being modeled. Finally, enabling processes includes strategies to search for information, numeracy skills, but also the ability to communicate detailed statistical information comprehensibly to diverse audiences.

2.2 Developing statistical literacy

The emphasis on skills, understanding of context, motivational dispositions, and critical thinking poses a challenge for the planning and designing of lessons. Statistical literacy is a demanding discipline from a didactic point of view, because it requires as prerequisite knowledge and skills from different subjects (Ben-Zvi & Garfield, 2004). According to Pfannkuch et al. (2018), statistical education should no longer be conducted exclusively through traditional approaches such as the theoretical classical or experimental frequentist approach. The classical approach involves teaching statistical concepts through fixed mathematical models and assumptions. The frequentist approach, on the other hand, is based on the interpretation of probabilities through the frequency of events, demonstrated through experiments and simulations. Instead, a modeling-oriented approach is recommended, focusing on the creation and application of models to represent and analyze real phenomena. This perspective promotes a practical and flexible application of statistical methods and better prepares learners for the complexities of the real world. So Pfannkuch et al. (2018) propose to use principles of mathematical modeling to teach principles of statistical literacy. Data-based modeling with functions represents one possible way to implement processes connected to statistical literacy into secondary education (Engel & Kuntze, 2011). Wild and Pfannkuch (1999) characterize modeling with data as a fundamental way of thinking in statistical contexts since the constant alternation between the real situation, the associated statistical data, and mathematics plays a central role. Before we characterize and describe data-based modeling in more detail, we first talk about the properties of mathematical modeling, because data-based modeling combines contents and processes of statistical literacy with mathematical modeling.

2.3 Data-based modeling with functions

2.3.1 Mathematical modeling

Greefrath (2018) characterizes mathematical models as abstract representations of real phenomena, problems, or situations that are described using mathematical concepts and tools. They are used to simplify and analyze complex systems by filtering out the relevant aspects of a problem and translating them into a mathematical form for a systematic analysis. This requires a simplification of the real world aiming to support understanding of the underlying structures, relationships, and behaviors (Kaiser et al., 2015). Modeling enables the conversion of complex problems into a mathematical structure that can be analyzed with mathematical techniques. However, the appropriateness and validity of a mathematical model may depend on the correct selection of variables and the assumptions made. The choice of modeling approach is largely influenced by knowledge about the context and pre-existing theories or concepts about the phenomena , as well as by the available data, and the analysis objectives. Modeling is a creative act and there is no universal method to generate a mathematical model, but rather various approaches that can be applied depending on the context (Greefrath, 2018).

The modeling cycle (see left part of Fig. 2) describes a prototypical approach to generating mathematical models. The entire process is frequently represented as a cycle (e.g., Blum & Leiß, 2007). Blum and Leiß (2007) describe mathematical modeling as a systematic process that follows an iterative approach. The description of the process in the modeling cycle with its individual phases follows the explanations by Blum and Leiß (2007) and Greefrath (2018). In the first phase, the problem needs to be identified and understood. This involves recognizing and formulating a clearly defined problem from the real world. This leads to the creation of a simplified and structured real model. This requires, like in the characterization of statistical literacy, to understand the underlying context, relevant characteristics, and their relationships. In the phase of model construction, the real problem is transformed into a mathematical model. This phase may involve the selection and definition of variables that represent the relevant aspects of the context, as well as describing their relations mathematically, for example by generating equations or functions that describe these relations. Mathematical methods, techniques, and tools can then be used to determine a mathematical result. The meaning of this result must then be interpreted in relation to the real-world problem. Here, the model becomes a tool for making statements about the real context. Often, the model needs to be validated to assess whether it describes the real situation in sufficient detail to draw valid conclusions. If there are doubts about the suitability of the model, adjustments need to be made to improve its appropriateness and validity. This may lead to further iterations of the modeling cycle.

2.3.2 Data-based modeling

Traditional perspectives on modeling often describe the generation of a mathematical model by matching characteristics of the context with a mathematical concept and then formulating a mathematical description that mirrors the contextual dynamics. The latter may comprise identifying a process as exponential growth, determining the initial value and growth rate over time from the context, and thus defining the model function type accordingly (structural modeling). When the relationships between relevant variables are unclear, especially if prior knowledge is limited and no established theory is available, a more data-driven modeling approach can be helpful. In such a data-oriented method, we begin by exploring the data without committing to a specific functional type (e. g., linear or exponential). Instead of fitting functions to the data by choosing from a range of possible types and adjusting parameters (e. g., initial value and growth rate for an exponential model), data-smoothing techniques may be more appropriate in complex situations (Engel, 2016) to illustrate the relationship between two numeric variables graphically.

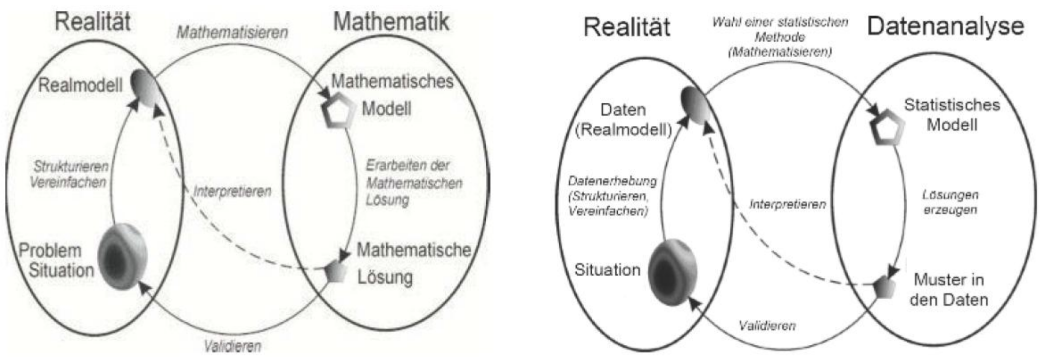

Fig. 2: Cycle of data-based modeling processes (source: Eichler & Vogel, 2013, p. 132)

Finally, by interpreting the resulting functional relationship within the context in which the data were collected, the data-oriented approach can yield deeper insights into the dynamics governing the relationship between the two variables, potentially pointing to a possible function type. Because patterns are discovered directly from the data, this approach is inherently exploratory. In this sense, data-based modeling with functions (Engel & Kuntze, 2011; Pfannkuch et al., 2018) aims to extract information from collected data to gain a better understanding of the underlying system from which the data arose, make predictions on the future behavior of the system, and optimize decision-making processes. To achieve this, one seeks to represent the relationship between two numerical variables through a model function (functional model; Eichler & Vogel, 2013; Engel, 2018; Vogel, 2014; Vogel & Eichler, 2014). If different functional models are compared, validation as well as interpretation may also draw on the properties of the selected model. For example, choosing a linear function or an exponential function allows either to draw different conclusions about the context and dynamics of the underlying system due to their differing functional properties (Engel, 2018; Vogel & Eichler, 2014), or to validate the model against assumptions that have been made about this system. Moreover, the derived functional model allows to make predictions, which can be used to derive interventions aiming to control the future development of the system (Engel, 2018). To obtain such a model function, in data-based modeling processes this is frequently done in a parametric way, i.e. a function type is selected and parameters specifying the concrete function are estimated from existing data. This estimation involves selecting parameters that ensure the function describes the observed data as well as possible, in a sense that needs to be specified. A classical approach for this is the method of ordinary least squares. Learning data-based modeling is not only a promising goal of school mathematics, but it also relies on and may also contribute to developing the entailed mathematical concepts. Structural modeling requires to use defining covariational properties of different function types—such as the constant relative change over equal intervals in exponential functions. Model interpretation digs into the meaning of the parameters describing the selected function type (e. g., intercept or slope in a linear function). Applying these properties in the context of data-based modeling may reactivate them but also contribute to practicing their application. Although students were generally aware of these properties, many had not seen them applied directly in a data-based modeling context. Therefore, we assume that a profound understanding of the covariational properties and parameters of the applied function types would affect the students' learning process (Vogel, 2014).

Secondly, data-based modeling is also about learning key statistical knowledge and concepts (Garfield & Ben-Zvi, 2005). For example, the concept of sources of statistical variation (signal-noise metaphor, Borovcnik, 2005) comes into play when the model function is expected to be only an approximation, not an exact interpolation of the data points—e. g. when balancing the fit of the model to the data with its interpretability and plausibility. Moreover, comparing different functional models aligns with a modeling perspective on statistical literacy. Different models can highlight different aspects of a phenomenon. Learners reflect on how the choice of model depends on the specific question at hand and the data available. Finally, by applying models to real-world issues, students may come to understand the role of statistics in various societal contexts. This involves recognizing how statistical models can help address social issues and how statistical analyses can shape discussions in politics and society.

2.3.3 Ordinary least squares (OLS) for model-data-fit

Within parametric regression models, OLS is the standard method for fitting a curve to data, whether the relationship is linear or non-linear (Engel, 2016). The residuals r_i with $i \in \mathbb{N}$ describe model-data-fit as the difference between the data points $(x_i, y_i)_{i=1,\dots,n}$ and the values predicted by a model function (Engel, 2018). When using an appropriate model function, the residuals should lack any structure and resemble a random process. The residuals are summarized into a single statistic. For a linear model $f(x) = mx + t$ with $m, t \in \mathbb{R}$, the sum $S(m,t) = \sum_{i=1}^{n}(y_i - mx_i - t)^2$ needs to be minimized. It is presented as an optimization problem depending on the variables m and t. This problem can be solved analytically (linear regression; see Eid et al., 2017). It can be shown that the estimator minimizing the sum of squares (SQS) $\sum_{i=1}^{n} r_i^2$ with the number n of data points has the smallest variance among all linear unbiased estimators, in accordance with the Gauss-Markov theorem (see Kockelkorn, 2000).

In some cases, it is possible to transform a non-linear model into a linear form, perform straight-line (linear) regression, and then convert the results back to the original scale. For example, if the model follows a power relationship $g(x) = ax^b$ with $a, b \in \mathbb{R}^+$, taking the natural logarithm yields the linear relation $ln\big(g(x)\big) = ln(a) + b \cdot ln\,(x)$. This makes it possible to apply linear regression techniques to the transformed data (cf. Vogel & Eichler, 2014). However, for certain functional models, such as logistic functions, a direct transformation to a linear form is often not feasible, and more heuristic approaches may be required (Engel, 2010). Alternatively, numerical optimization methods (such as the Gauss-Newton algorithm, see Nocedal & Wright, 2006) can be used to minimize SQS for non-linear models without transformation (Engel, 2018). While this iterative procedure requires advanced numerical analysis skills, a more accessible approach for secondary school students is to use software that calculates SQS. Students can then manually adjust parameters (for example, with sliders) to find the best fit.

When applying this with secondary school students, it is probably advisable, initially, to reduce complexity by not linearizing the different function types but instead directly using SQS without transformation. We concentrated on the least-squares problem across different types of functions, paying particular attention to the importance of squared residuals. Although alternative error metrics such as absolute values, weighted squared errors, or relative errors can be used in advanced analyses, their introduction would exceed the goals of the LMUmathlab and should therefore take place in later learning phases.

2.4 Process frameworks for working with data

Apart from specific concepts and approaches, developing statistical literacy and, more specifically teaching data-based modeling, also have a strategic component that intends to support students' self-regulated problem-solving. There are now several frameworks that describe processes of working with data. They also serve as a pedagogical construct for designing a learning trajectory and as a theoretical construct for analyzing students' reasoning.

A student-oriented presentation of Wild and Pfannkuch's (1999) PPDAC cycle in the Census at School project (2021) describes each phase as "detective work." In the problem phase, students identify the phenomenon and formulate a relevant question, understanding that data is essential to address it. In the second phase, they plan how to collect suitable data. Phase three covers data collection and processing, while phase four involves analyzing the data with statistical methods and models. Finally, in phase five, conclusions are drawn by interpreting results within the context.

The process in the PPDAC cycle shows many similarities to the cycle of mathematical modeling. Eichler and Vogel (2013) as well as Wassner and Proemmel (2022) specified the mathematical modeling cycle to data analysis differentiating the following phases: Clarifying the phenomenological background, considering the data in different representations, choosing a function type, and estimating parameters for a specific function, as well as validating the model by checking its predictions against prior assumptions, interpreting it, and drawing conclusions.

The framework by Lee et al. (2022) describe phases of a data investigation process on a survey from a sample of data scientists: framing a real-world problem, considering or gathering data, structuring and visualizing data, identifying and analyzing models that address the initial problem, and finally communicating results and proposing actions.

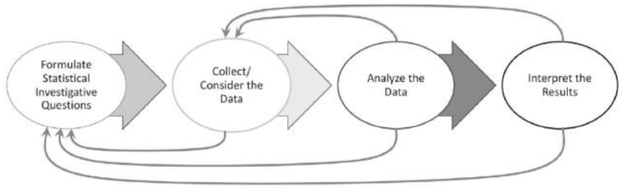

Fig. 3: GAISE framework for a statistical problem-solving process

Bargagliotti et al. (2020) describe a statistical problem-solving process within the Guidelines for Assessment and Instruction in Statistics Education (GAISE), structured into four phases (Fig. 3): formulate statistical investigative questions, collect or consider the data, analyze the data, and interpret the results. Formulating questions highlights the importance of recognizing variability in data and ensuring a thoughtful data collection process. In the data collection phase, discussions include data gathering methods, types of variables, and potential limitations and generalizability. During data analysis, variability is explored and described using graphical representations and numerical summaries. Finally, in the interpretation phase, conclusions are drawn, taking the inherent variability of the data into account.

The detailed examination of various frameworks for working with data reveals a consistent emphasis on iterative processes. Also the framework of Bargagliotti et al. (2020) describes circular dynamics indicated by the reverse arrows. Insights, newly built knowledge and identified new needs gained within one cycle can lead to further cycles. However, observation studies show that a cycle is not processed linearly, phase by phase, but students go back to previous phases until the results are satisfactory to progress (Borromeo Ferri, 2007; Ostkirchen & Greefrath, 2022). The frameworks emphasize the iterative nature of data processes, which means that learning experiences should be designed flexibly so that students can move between phases. The described cycles underline the necessity for dynamic problem-solving, where students independently adjust their models and conclusions. The frameworks, each with their unique phases—from identifying and framing problems, through data collection and analysis, to interpreting and communicating findings—underscore the nuanced and dynamic nature of engaging with real-world data. Furthermore, the importance of interpretation and context is highlighted, so the student laboratory should not only teach the technical aspects of modeling but also foster understanding of the underlying phenomenon and its connection to the real world.

2.5 Choice of contexts

During modeling, the sensible interchange between real-life situations and mathematics plays a crucial role. Through building their own models, students are able to observe and analyze real-world phenomena from a mathematical perspective (Engel, 2016; Greefrath, 2018). The associated goal is to better understand or explain our own environment by using mathematical tools (Blum & Leiß, 2007; Greefrath, 2018). Pfannkuch et al. (2018) describe this as an interconnected approach. This approach aims to facilitate learning not only about mathematical and statistical concepts and tools but also about the associated real-world phenomena. In this context, it is often criticized that mathematics teaching rarely introduces questions that are relevant in everyday life (Gal et al., 2022), which limits the fulfillment of modeling objectives. In terms of teaching practice, this implies taking an authentic context that is interesting and relevant to the learners as a starting point (Gal, 2022; Wassner & Proemmel, 2022). Wassner and Proemmel (2022) describe the use of contexts as a crucial factor for successful teaching, as students need to overcome the "Why should I care?" attitude by finding topics that genuinely interest them. In this regard, Gal (2019) emphasizes the central importance of question-posing. He suggests that specific questions related to a context dictate the students' "need to know". Contexts and the questions posed about them seem to play a crucial role in modeling and statistical literacy (Pfannkuch et al., 2018).

3. General goals and relations to existing didactical concepts

The development of the LMUmathlab aimed to complement existing didactical concepts for data-based modeling by focusing on specific goals.

First, due to the restricted time in the student laboratory, we aimed at an initial introduction into data-based modeling in the sense of a "first encounter", with the concept of model-data-fit. This complements existing approaches, which often comprise much longer time scales to offer an in-depth treatment also of additional topics such as comparing different smoothing methods (Engel et al., 2008), exploring complex datasets or ways presenting analysis results (Wassner & Proemmel, 2022).

Second, we intended to focus on established function types in secondary mathematics, so that the student laboratory can easily complement ongoing mathematics instruction of these function types. More specifically, we focused on the covariational properties of these function types, since we see them as central conceptual knowledge about the function types. While these properties are often checked in an exploratory way based on existing data (e.g., Vogel & Eichler, 2014), we aimed at emphasizing the possibility to relate them to the presented context situation itself. Thus, beyond model-data-fit, we aimed to convey knowledge and strategies to select models based on assumptions about the context (structural modeling) and the interpretation of functional models and their parameters.

Third, due to its (lasting) importance in statistical practice and the possibility to address the signal-noise-metaphor with familiar mathematical contents, we intended to focus on parametric regression (Biehler et al., 2011), as opposed for example to local stochastic processes in other concepts (Engel et al., 2008) or works that primarily address very basic descriptive statistics such as measures of central tendency or variability (Sproesser & Brühne, 2022).

Fourth, we intended to provide insights into the interplay of context-driven, theory-related structural modeling processes and data-driven modeling when choosing the functional model type, addressing typical and complementary approaches towards data-based modeling highlighted by Engel (2016).

Fifth, we decided to focus primarily on real data, because we considered simulated data to have potential primarily to address deeper issues such as the role of model complexity (e. g., number of parameters) for model fit (e.g., Engel, 2016).

Sixth, regarding context selection, we followed existing works (e.g., Pohlkamp, 2022; Sproesser & Brühne, 2022) that focus on data related to environmental issues, keeping in mind that other contexts, such as civic education (Wassner & Proemmel, 2022) or biological topics (Vogel & Eichler, 2014), may be necessary to address a broader range of interests (Leiss et al., 2024) and to build widely applicable data literacy.

Lastly, we deliberately decided to explicitly introduce a basic statistical modeling technique (OLS) as it also seems to be the case in Engel et al. (2008), instead of less guided, inquiry-based approaches, which are used for example in the "Gender Pay Gap" project by Wassner and Proemmel (2022).

4. Design principles of the LMUmathlab

Statistical literacy encompasses a range of knowledge necessary for understanding, interpreting, and critically evaluating statistical information. Data-based modeling aims to create functional models that represent the relationship observed in data (Engel, 2018; Vogel & Eichler, 2014). The processes involved in working with data can be mapped using the principles of mathematical modeling. It provides a framework for understanding complex systems and making predictions. According to Engel and Kuntze (2011), modeling with data represents one possible approach building statistical literacy. So, treating data-based modeling in school may open opportunities to convey knowledge and dispositional aspects of statistical literacy (Engel & Kuntze, 2011; Gal, 2002; Pfannkuch et al., 2018).

More generally, Bakker (2018) argues that educational research should investigate how learning *can or should be,* and proposes to work towards design principles that can guide future development of learning environment as theory-, experience- or evidence-based prescriptive scientific knowledge. So, the question arises what specific design principles should be applied to a student laboratory focusing on data-based modeling.

Designing a learning environment—and more specifically a student laboratory—encompasses decisions about content and its presentation, as well as the orchestration of students' work with this content. Along these lines, we distinguish between specific *content design principles* (CDP), that align with the goals of fostering competencies, and *methodical design principles* (MDP), that describe the orchestration of student work (Table 1). In the sequel, we briefly introduce the CDP's we derived from the presented background on statistical literacy and modeling with data.

4.1 Deriving content design principles

The CDPs collectively represent more comprehensive criteria for designing an effective learning environment and thus extend beyond the content aspects of data-based modeling.

Tab. 1: Content and methodical design principles implemented in our student laboratory

Content design principles	
CDP 1	Introducing a process framework.
CDP 2	Activating relevant mathematical knowledge.
CDP 3	Explicating the phenomena underlying statistical concepts.
CDP 4	Using and connecting statistical and mathematical concepts.
CDP 5	Implementing authentic contexts.
Methodical design principles	
MDP 1	Implementing a part-task approach.
MDP 2	Supporting collaborative learning with a collaboration script.
MDP 3	Providing content-related scaffolding.
MDP 4	Leveraging digital tools.

Well-founded instructional concepts emerge when relevant didactic considerations are combined with findings from teaching-learning research and general psychological theories on learning and teaching (Reinhold, 2019).

(CDP 1) Explicitly introducing a process framework, an iterative process, that can guide students' planning, monitoring, and reflection of data-based modeling activities (Zöttl et al., 2011): The complexity of data-based modeling underscores the importance of providing students with a systematically approach towards data analysis and interpretation. Pfannkuch et al. (2018) summarizes that the understanding of the specific reasoning involved in statistical modeling processes, as well as the theoretical and pedagogical approaches, are still at the beginning. Thus, the question arises as to how such a process framework for data-based modeling processes can be designed.

(CDP 2) Activating and integrating relevant mathematical knowledge: To effectively engage in data-based modeling, students must draw upon and activate their mathematical knowledge, particularly understanding the covariational properties of functions. (CDP 3) Explicating the phenomena underlying statistical concepts: This includes that students can experience the meaning of statistical concepts such as model-data-fit in terms of the difference between "signal" and "noise", that are often oblique in later complex practices. We decided to introduce these ideas based on a specific instantiation (functional models and SQS). For example, explicitly seeing what the SQS as a measure of model (mis-)fit (noise) means for a specific functional model and a specific dataset, how it is calculated, and how it changes when the function is changed allows to experience its meaning, before the corresponding concept used as an object of its own and extended by further, alternative instantiations (other model types, other noise functionals).

(CDP 4) Connecting statistical and mathematical concepts by reduced, but authentic practices and techniques: For example, using a manual least-squares estimation approach allows to explicitly experience the role of the SQS in determining a model function (cf. Vogel & Eichler, 2014). Thus, students become actively engaged in using the concepts in the application of the concepts—in our case applying covariational properties, statistical variation, fitting parameter with methods like OLS, and compare models—when encountering or practicing statistical practices and techniques.

(CDP 5) Implementing authentic contexts through an interconnected approach: This involves using real-life situations as a basis for modeling activities, thereby making learning more relevant and interesting for students (Leiss et al., 2024).

4.2 Deriving methodical design principles

In the field of research on student laboratories Woithe et al. (2022) conclude from their study that the impact of student laboratories depends on the quality of the learning opportunities. Rodenhauser (2016) adds that students should feel they are in an open learning environment that allows for choices. The challenge in designing such environments is not to leave students alone and without support when solving problems. Rather a structured and instructive learning environment is required, in which learners do not feel explicitly guided, but receive support when necessary. In the following, we describe specific characteristics, that have been described as effective for learning environments—especially also for student laboratories (e.g., Euler &

Schüttler, 2020; Mandl & Friedrich, 2006; Ryan & Deci, 2020; Vogel, 2014). From this, we then derive specific methodical design principles for our student laboratory.

Implementing a part-task approach (MDP 1)

To reduce complexity in learning, tasks can be broken down into meaningful subtasks, allowing learners to work on individual elements before tackling the entire task (Hasher, 1971). According to Van Merrienboer et al. (2003) in cognitive load theory, this decomposition reduces the cognitive load compared to that of the whole task. Conversely, whole-task approaches emphasize integrating skills to foster a holistic understanding. Wickens et al. (2013) found in their meta-analysis that part-task approaches are especially effective when activities are highly complex and divisible into meaningful components, as in mathematical process frameworks, and when learners have limited experience but understand the overall task goal. Implementing part-whole principles in introducing a mathematical process framework (MDP1) can involve (i) presenting the full process first (e. g. in a worked example), (ii) introducing each step of the process framework individually, and (iii) enabling whole-task practice through a complete data-based modeling transfer task.

Supporting collaborative learning with a collaboration script (MDP 2)

Collaborative learning, where students work together to solve problems and build knowledge, enhances academic performance, problem-solving skills, motivation, and positive learning attitudes (e.g., Chen et al., 2018). Despite its advantages, challenges like unequal participation and superficial discussions can limit its effectiveness (e.g., Le et al., 2018). Research suggests that structured guidance, such as collaboration scripts, helps improve group dynamics and learning outcomes (Chen et al., 2018). Collaboration scripts include specific elements like role assignment, discussion prompts, sequences, and instructions, facilitating interaction and effective idea exchange, especially when combined with content-specific scaffolding (Vogel et al., 2017). By implementing these, cooperative processes in task engagement can be stimulated. Thus, using collaboration scripts in collaborative learning (MDP 2) offers a methodical approach to address common challenges and enhance educational outcomes.

Providing content-related scaffolding (MDP 3)

Wood et al. (1976) describe scaffolding as a process that enables learners to achieve goals beyond their unassisted efforts by adjusting activity demands to match their capabilities. Effective scaffolding aligns cognitive requirements with learner characteristics (Kollar et al., 2018). Methods for implementing scaffolding include questioning, arguing, explaining, collaborative scripts, message starters, prompts, instructions, and hints (see van de Pol et al., 2010; Vogel et al., 2017). Research confirms the effectiveness of scaffolding, particularly when it complements collaboration support (van de Pol et al., 2010; Vogel et al., 2017). Content-related scaffolding can be provided by tutors offering minimal support when students reach an impasse in their self-regulated learning. To this end, it is helpful if tutors analyzed the learning activities, anticipated student responses and prepared possible (verbal) scaffolding moves (e.g., Hammer & Ufer, 2023) beforehand. Moreover, tutors can monitor, if the main concepts and ideas for each phase of the process framework (cf. section 5.2) was covered, and provide stimulating questions if necessary to direct the discussion to missing aspects.

Leveraging digital tools (MDP 4)

Digital tools may enhance complex modeling in classrooms (Greefrath et al., 2018) by enabling visualization and linking multiple representations—graphical, symbolic, numerical, and verbal (e.g., Burrill, 2014). Furthermore, representing function graphs dynamically through dynamic geometry software (DGS) allows students to explore and investigate relationships between function parameters and data, and gives them the opportunity to create, represent, evaluate, and refine models that describe these relationships (Greefrath, 2018; Vogel, 2014).

Another promise of digital tools is to outsource complex calculations (Greefrath, 2018), such as calculating SQS, thereby shifting the focus to the core mathematical ideas, such as model-data fit. A spreadsheet or a prepared DGS sheet can automatically take over such calculations. As a result, students can directly utilize concepts such as variance in functional models without getting heavily involved in the calculations themselves—but still have the opportunity to unpack the calculation by analyzing the sheet. Outsourcing complex calculations to digital tools enables a deeper immersion into mathematical content to enhance understanding of mathematical structures and relationships. Moreover, it allows students to engage with mathematical content that may not have been possible without the use of digital tools (Engel, 2018; Vogel, 2014).

5. The LMUmathlab: Designing a student laboratory for data-based modeling

As Pfannkuch et al. (2018) raises the demand to develop theoretical and concrete pedagogical approaches towards statistical modeling one aim of initiating the LMUmathlab was to design a student laboratory for data-based modeling. We present the design of the LMUmathlab in terms of practical procedures, content design, and learning orchestration.

5.1 Procedure design: Structural organization

The LMUmathlab was aimed at upper secondary students from schools in the high attaining school track in Germany. In total, 504 school students have already participated (M_{age} = 16.3; SD = 0.99; male= 245; female = 254; diverse = 5). The schools applied to participate, and we applied no targeted selection process. Participation took place in class groups, but was voluntary for individual students, and required written consent of the participants. The student laboratory, created during the Covid-19 pandemic, was presented an online system that implemented a collaboration script to guide the whole process (MDP 2), so that the laboratory could be conducted in distance learning as well as in a face-to-face format. In the distance variant, students logged in from their homes, while in the face-to-face variant, all students were provided with their own devices.

The laboratory, was connected to a mathematics education seminar for preservice teachers, covering both the content aspects (data-based modeling) and the methodological aspects (implementation of specific design principles) of the LMUmathlab, with a specific focus on scaffolding and tutoring in mathematics education. The seminar participants tutored the student groups in the student laboratory, applying the course contents and gaining initial practical teaching experience. For their participation, the students received course credits.

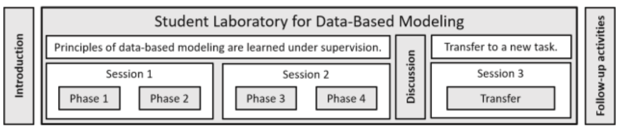

Fig. 4: Process of the student laboratory

The general procedure (Fig. 4) of the student laboratory consisted of an introduction and three sessions that built upon each other. Before visiting the student laboratory, students received an online introduction that presented the topic of data-based modeling[1]. The first two sessions served as the learning phase, where students were introduced to the principles of data-based modeling with concrete example problems. Students worked in self-selected small groups of three to four students and were supported by a mathematics pre-service teacher. In each learning phase, the student groups were tutored by a different pre-service teacher to make the process less dependent on the supervising person. Following the learning phase, a discussion took place in a plenary setting, where individual groups presented their results, which were then collaboratively discussed by the entire class. The third session was a transfer phase, where students transferred the learned concepts and techniques to a new task, without support longer by a tutor. Additionally, the teachers of the visiting classes were provided with two data-based modeling supplementary tasks to work on during regular classes after visiting the student laboratories, to ensure follow-up activities (Lewalter, 2020).

5.2 Content design: Synthesizing and reducing a process framework

Traditional student laboratories tend to focus on curriculum topics (LernortLabor, 2023). However, this approach poses challenges in terms of organization, planning, and scheduling, as participating classes can only visit the laboratory at specific times during the school year. Therefore, we opted for data-based modeling, which is scarcely included in the Bavarian curriculum but directly builds on the content covered there.

To structure students' work, we created a simplified data-based modeling process framework (Fig. 5) for our student laboratory based on the existing frameworks for working with data (CDP 1). All these frameworks highlight the search for a pattern in a dataset, that is to be represented by a mathematical model. We focus on situations in which a bivariate relation between two continuous variables needs to be modeled with a parametric model function.

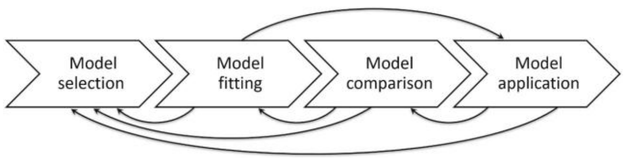

Fig. 5: Data-based modeling process framework

1. Model selection: Our prototypical data-based modeling process starts with the selection of a specific function as a model for the context (model selection). This includes arguing why a specific model function type can be selected or excluded for a given context. At this point, this selection is only made on the basis of knowledge about the context

(e. g., by analyzing whether a function type's covariational fit the context) or on the data (e. g. its shape as visualized in a coordinate system). This results in one or more competing model function types.

2. Model fitting: The relationship in the data is then represented by a concrete model function for each selected model function type. Parameter estimation yields an optimal function of a specific type to describe the relationship numerically (model fitting). Methods to estimate a suitable function may range from informal processes—such as visually fitting a graph to the data—to more sophisticated modeling techniques—such as ordinary least square techniques implemented as analytic or approximative algorithms.

3. Model comparison: If competing model functions were selected in the first step—which is usually the case, comparing them may provide evidence about the (relative) validity of the resulting models. This connects directly to the model selection in the first process phase and leads to an optimal model being identified that maps the trend in the data set as good as possible (model comparison). Most fundamentally, drawing on the covariational properties of the optimal model type conclusions can be drawn about the type of system that underlies the observed relationship (e. g., linear or exponential growth). Based on this, the model parameters can be interpreted and compared as properties of the system (e. g., speed of growth).

4. Model application: Finally, all frameworks include applying the model to make statements about the specific problem or context (model application). More practically, the model may be used to predict or interpolate values that are not included in the data, or to draw conclusions about an average, to be expected value.

The arrows in the process framework illustrate that repeated transitions between individual phases are likely to occur dynamically in the overall process, rather than in a strictly linear order. Additionally, validation steps may take place throughout the process (Borromeo Ferri, 2007; Ostkirchen & Greefrath, 2022), which can impact the model selection. For example, if students realize that the selected and fitted model, they intend to apply does not adequately reflect the context, they may choose a different model function or they may reassess and compare the fitted models.

Many frameworks include phases of collecting, preparing, and presenting data. For our student laboratory, we excluded these elements based on Wassner and Proemmel's (2022) guidelines, which recommend using existing public data to help students focus on real data analysis rather than data collection. Collecting and preparing data is time-intensive and difficult to organize within the lab's constraints. Instead, we prioritized the more conceptual aspects of model selection, model fit, and model comparison over the often more practical issues of generation data collection instruments, data collection itself, and data preparation. However, it is undebated that curricula for statistical literacy should aim at conveying extended process frameworks, also including these activities.

5.3 Content design: Selecting contexts

Incorporating real datasets from the fields of environment and sustainability, we aimed to engage upper secondary school students with contexts that are relevant to them. The selection of contexts and the design of questions were planned to foster a connection between students and the real-world issues they study, embodying the interconnected approach (CDP 5). This intends to motivate students for both, to solve the problems entailed in the contexts and to engage deeply with principles of data-based modeling—but also to convey an idea of the role statistical methods play when solving real problems.

We introduced students to a variety of environmental issues through specific contexts: the increasing presence of plastic in our oceans, the feasibility of meeting energy demands exclusively with renewable sources by 2050, the future of the Aletsch Glacier, and the global efforts to limit warming to 2 degrees Celsius. Each context was accompanied by a question designed to provoke thought, inspire investigation, and make the learning process as authentic as possible (first initial question). For example, we explored the prospect of oceans containing more plastic than fish, questioned the realism of transitioning entirely to renewable energy sources within the next few decades, thought about the existence of the Aletsch Glacier by the century's end, and (very roughly, though) evaluated the prospects of achieving a specified global warming limit. An overview of these contexts, as featured in the LMUmathlab, is detailed in the Supplemental Materials. These contexts were chosen not only for their environmental importance but also for their ability to resonate with the students' different interests, hoping to inspire a sense of urgency and a desire to engage with answering the questions. Across all contexts, we presented students with the overarching goal to learn to investigate how certain data has developed over time (second initial question).

5.4 Learning orchestration

5.4.1 Orchestrating contexts and contents

In designing the learning setting for the student laboratory, particularly for beginners in the field, it was essential to recognize that navigating through the entire data-based modeling process independently might be overly ambitious. Therefore, we adopted a part-task approach (MDP 1), which allowed us to break down the complex modeling process into more manageable phases. Four learning phases were designed, one for each process phase from the data-based modeling framework. Each learning phase was then further divided into a sequence of activities (MDP 2: sequencing).

To enrich the learning experience, we structured the process so that students are confronted with a new context in each phase. For example, in Phase 1, students would delve into model selection with a focus on glaciers, then move to model fitting in Phase 2 within the CO_2 context, followed by model comparison in Phase 3 centered around energy consumption, and finally, model application in Phase 4 with plastic as the context. Each group received a different assignment of contexts to learning phases, so that the discussion after all four phases could draw on a range of different experiences. Switching to new contexts also allowed us to engage students in new explorations, based on common results from prior process phases in each learning phase. Additionally, this part-task approach offered the flexibility to skip some contents of prerequisite phases—e. g., the strict necessity to fit several models before using them in model comparison (MDP 1).

The aim of this design was twofold: firstly, to ensure that all students were exposed to each context once, providing them with a spectrum of examples for the application of data-based modeling; and secondly, to allow students to experience the same statistical concepts in different contexts, allowing to convey a general view on the concepts (CDP 3) and the process (CDP 1) of data-based modeling.

5.4.2 Learning phases in the LMUmathlab

In the following we describe the four learning phases in the student laboratory. The contents for each learning phase were embedded in one of the four contexts. For better comprehensibility, we always explain the activities in the CO_2-context; the activities in other contexts were analogous (see Supplemental Materials). We also describe to what extent the use of digital tools supported the work of the students in the individual activities (MDP 4).

Phase 1: model selection

The first learning phase of the LMUmathlab focused on model selection. Especially with real datasets, it is usually challenging to directly identify an exact functional model (Engel, 2018). Therefore, the primary goal of the first learning phase was not to select a specific model function for a dataset but to consider whether certain model types could be excluded (CDP 4). Here, the students were focused on understanding the context and the entailed variables and evaluating whether the relationships between these variables aligns with the covariational properties of the three functional models specifically focused in the student laboratory: linear functions (including proportional functions), exponential functions, and power functions.

(1.1) The first activity in this learning phase aimed to deepen understanding of the three types of functions by exploring their unique properties. For this, an interactive app was created where students sorted different function representations. The task encouraged group discussions (MDP 2) to identify differences, with a special focus on the covariational property of the functions (CDP 2).

(1.2) In the second activity, to address the challenges students face with the covariational property (e.g., Lichti, 2019), they were tasked with applying this concept to real-life scenarios introduced in the initial video (CDP 4). For example, using the function $f(x) = 0.001 \cdot x^3$ to depict the relationship between wind speed and power output of wind turbines, students explained how a doubling of wind speed leads to an eightfold increase in power, illustrating the covariational property. The assignment involved completing sentences that clearly articulated these relationships across various contexts (MDP 3).

(1.3) Activity 3 focused on model selection, specifically the exclusion of models. In the first step, the student groups were asked to exclude one function type—linear function, exponential function, or power function—that they believed would not be suitable for representing the CO_2 context. Students collectively made initial selections within their group (MDP 2) through an applet (MDP 4). To ensure that this initial hypothesis was primarily based on structural considerations, drawing solely on the students' prior knowledge and their interpretation of the given context description, access to the CO_2 emission data was provided only *after* this decision was made. After access to the data, they deliberated as a group on their initial choice (MDP 2), deciding whether to stick with it or to exclude a different function type based on the newly presented data. The applet then offered feedback on the choices made (MDPs 3 & 4).

Phase 2: model fitting
Learning phase 2 aimed at learning a strategy and related concepts to fit a model to the trend in a dataset. This included a basic version of the OLS method which comprised manually adjusting the parameters of the model function type, as well as introducing the related statistical concepts (CDP 3), such as the SQS as a measure of residual statistical variation.

(2.1) In the first activity, students adjusted a function type that was not excluded in learning phase 1 by manipulating its parameters using a provided applet (MDP 4) while visually evaluating how well their model aligned with the data. Given that each student worked with their own tool for this exploration, we expected that they would develop different functional models for a single model function type. We anticipated that the groups would conclude that they cannot reasonably decide between these different models that visually fit the dataset equally well.

(2.2) This intended to trigger a need to introduce and define a measure of goodness. The principle of Ordinary Least Squares (OLS) was introduced (CDP 3) using a short explanatory video[2].

(2.3) In the third activity, the students were then asked to use OLS to fit their functional model type model to the data as good as possible (CDP 4) For this purpose, the SQS was added to the applet from (2.1, Fig. 6). The students were prompted to find a strategic approach to determine the smallest possible SQS.

The fitting process targeted by tutors was based on a VOTAT strategy (Vary One Thing At a Time, Molnár & Csapó, 2018; Tschirgi, 1980), where students adjust one parameter at a time to find the model's optimal fit using OLS. They first fix one parameter, optimize the other to minimize SQS, and then adjust the fixed parameter in search of a smaller SQS value. The process continues, alternating between parameters, until they identify a local minimum SQS (CDP 3).

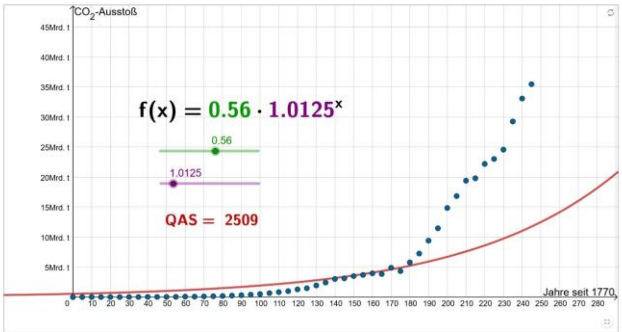

Fig. 6: Applet for model fitting using QAS (German for SQS)

Phase 3: model comparison
Learning phase 3 focused on comparing functional models of different types, that had been optimally adjusted using SQS.

(3.1) In the activity, the students were provided with a diagram depicting two OLS-optimal models from different function types—the two that could not be excluded for structural reasons in 1.3. For one of the models, the specific SQS was given, while for the other model, the students were required to determine the SQS using a spreadsheet (CDP 3). This allowed students to compare the model-data-fit using SQS as one possible criterion. We expected students to consider that a lower SQS might suggest a better fit between the model and the data (CDP 4). However, SQS alone is not an absolute measure of model quality and should not be used as the sole criterion for model comparison. It should always be considered alongside the specific initial question, underlying assumptions, and the chosen model type—as the number of function parameters complicates interpretability. Additionally, validating the usefulness of the chosen model is essential to ensure its appropriateness.

(3.2) The second activity was designed to deepen students' understanding of how functional models can be used to describe how a system changes over time (CDP 4), answering the first initial question in each context. This involved evaluating three descriptions of covariational properties, each interpreted in the corresponding context. The statements described linear growth, exponential growth, and a power relation. For the CO_2 emissions context, students received statements each describing one of the covariational properties in the context (Fig. 7). First, the students should determine how these statements came about. They were asked to transform the algebraic expression of the functional models so that they could match the covariational properties (CDP 2). The determination and interpretation of the covariational properties of the functional models should help in model comparison by allowing students to discuss the temporal dynamics. By examining how each model depicts the development of CO_2 emissions over time they could assess the appropriateness of each model. This discussion of temporal dynamics may be crucial for understanding which model more accurately represents the real-world trend in CO_2 emissions and, therefore, is more suitable for making future projections.

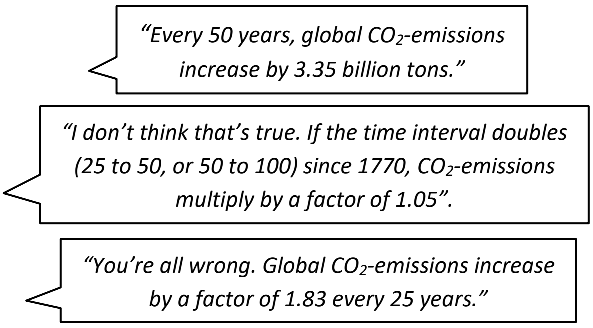

Fig. 7: Statements in model comparison

Through this activity, students not only performed mathematical calculations but also engaged in critical thinking about the relevance and applicability of different models to real-world contexts. This process should help them understand the importance of comparing appropriate models based on both mathematical fit and contextual interpretation.

Phase 4: model application

In learning phase 4, the focus shifted to applying the developed functional models. This learning phase primarily concentrated on two objectives: first, to utilize the model to tackle a concrete question relevant to the context and second, to summarize the phases of modeling in a report.

(4.1) The second activity focused on investigating the initial question in the CO_2 emissions context, whether we can achieve the 2-degree target. The task was framed based on reports from researchers (see Supplemental Materials), stating that from the years 2000 to 2050, no more than 1000 billion tons of CO_2 can be emitted to reach the 2-degree target. Based on the identified model, the students checked this criterion by generating a spreadsheet-calculation (CDP 4), using data on prior years, and the model's predictions. Furthermore, students were asked to evaluate the predictive accuracy of the developed model to estimate future CO_2 emissions. They needed to discuss in their own words how reliable the model might be for future projections. Students were also encouraged to describe factors that might influence CO_2 emissions and discuss the realism of the model's long-term predictions in the context of possible interventions that could alter the trend. This involved assessing the model's utility and validating its appropriateness by interpreting its properties and their relevance to the initial question and the broader context.

(4.2) After addressing the initial questions, each group prepared a report based on their context of phase 4. In this report, they summarized their journey through the data modeling process, highlighting key findings and insights gained across the four learning phases (CDP 1). The core of the activity was to reflect on and document the steps taken to model CO_2 emissions (or another context), the model comparison process, and the implications of their findings. Specifically, groups were encouraged to articulate how their model was constructed and adjusted, to analyze the relevance of their results in relation to the posed initial questions, and to discuss any limitations of their model or external factors that could influence its appropriateness. This task not only served as a comprehensive review of the context's scope—from initial question to final analysis—but also prepared the students to share their conclusions with peers, fostering a deeper understanding of the subject matter through collective discussion in a plenary session (MDP 2).

5.4.3 Supporting collaborative learning

To facilitate student collaboration throughout the student laboratory, a dedicated online collaboration script was designed, that guided student groups through collaborative work on the learning phases and activities. This script sequenced all contexts, questions, tasks, and applets.

Moreover, roles were explicitly assigned within each group to structure the collaborative process effectively (MDP 2: role assignment). Responsibilities such as group manager, time manager, results manager, and reflection manager were introduced and assigned to individual students. This strategy ensured that each member had a specific focus, contributing to the group's overall productivity and engagement. Role assignment facilitated a balanced distribution of tasks, ensuring that all students actively participated and that the group functioned cohesively towards its goals.

Fig. 8: Example of a guiding question (Translation: How can the parameters be determined so that the graph of the function reflects the trend of the data as accurately as possible?)

At the start of each activity, groups were presented with discussion prompts in the form of guiding questions (MDP 2: discussion prompt). These prompts served to stimulate critical thinking and focused discussion, guiding students towards the objectives of the activity (Fig. 8). By framing the tasks with specific questions, the script encouraged meaningful dialogue and exploration of concepts, ensuring that discussions remained on topic and were productive.

Each learning phase of the collaboration scripts included clear sequences (MDP 2: sequences), communicated through the guiding questions and the script's overall structure. Our aim was to ensure that clear goals were set from the beginning and that all students worked towards the same objectives. This also helped the time manager within each group to monitor progress and ensure that tasks were completed within the assigned periods—in this case, ten minutes. Implementing sequences helped maintain the pace of work, preventing any single task from overshadowing others and ensuring that the group could move through the activities efficiently.

For most of the activities, groups were given summarization instructions to reflect on their work, synthesize their findings (MDP 3: feedback), and prepare a coherent presentation of their results (MDP 2: summarization instructions). This not only reinforced the learning objectives but also encouraged students to critically assess their work and articulate their conclusions effectively. Summarizing served as a key moment for reflection and consolidation of knowledge, enhancing the overall learning experience. The results manager primarily entered the group's collaborative results into the text fields in the script. However, these text fields were designed to synchronize simultaneously for all students, allowing them to work together as a group on their results.

5.4.4 Providing content-related scaffolding

The content-related scaffolding in the student laboratory aimed to balance autonomous work among student groups with necessary support by tutors. Creating a supportive classroom atmosphere seems to be crucial to foster and sustain learners' willingness to actively engage with the subject matter. However, due to the heterogeneity of the student groups and the complexity of the topic, it was expected that not all students would be able to work fruitfully on the tasks without external support.

Each group was supported by a pre-service teacher (MDP 3: guided assistance), to provide on-demand scaffolding. Pre-service teachers were informed about their roles during the phases as a facilitator of learning rather than as direct knowledge provider. Therefore, unless any (content-related) problems arise, they should not intervene in the students' working process. This aligns with the design principle of providing dynamic, responsive support, adapting to the unique needs and questions that arise within each group (MDP 3). The students had learned strategies to offer both content-related help and affective support, promoting a constructive learning environment. By focusing on minimal intervention, the tutors encouraged groups to maintain their autonomy, thereby fostering a constant engagement with the material and enhancing problem-solving skills.

Structured feedback session concluded each learning phase, comprising first an internal feedback phase, facilitated by the reflection manager, where group members reflected on their collaborative process and its outcomes. This internal discussion aimed to help students recognize their collective achievements, strengths, and areas needing improvement or further information. Followed by this, the pre-service teacher provided external feedback, that was adaptively based on the group's reflection in the previous discussion. This feedback aimed at providing additional perspectives and guidance, focusing on clearly outlining the goals to be achieved, summarizing the progress made so far, and specifying what actions should be taken to facilitate further progress (Hattie & Timperley, 2007). This comprehensive approach to feedback—blending internal reflection with detailed external feedback from the pre-service teacher—was designed to support both individual and group learning processes, emphasizing the value of teamwork and using feedback as a driving force for continuous development and achievement of goals.

6. Looking back

The need to integrate data science education into school curricula has been emphasized by researchers (e.g., Biehler & Schulte, 2018). According to Pfannkuch et al. (2018), the research landscape shows that research on both, students' understanding of the underlying logic of statistical modeling processes and on theoretical and pedagogical strategies to foster modeling skills, are still in early stages. To address the identified research gaps our project focused on designing a comprehensive data-based modeling approach within a student laboratory setting. The primary objective was to introduce students to the foundational concepts and processes involved in data-based modeling, explicitly connecting to their school-based mathematical knowledge and skills.

We will discuss the following aspects, also drawing on first experiences from the implementation of the student laboratory (partially published in Mohr & Ufer, 2023): (1) The implemented mathematical and statistical concepts in relation to the objective of analyzing the potential of data-based modeling for enhancing statistical literacy. (2) The suitability of the developed process framework and potential for further development. (3) A reflection of the presented design in terms of procedural design and learning orchestration. This includes a review of the laboratory's structure and its alignment with the process framework, as well as the potential benefits and challenges of this approach.

6.1 Mathematical and statistical concepts

In evaluating the implemented mathematical and statistical concepts within the context of enhancing statistical literacy through a data-based modeling approach, several observations emerge from our first experiences. This discussion centers around three main areas: the use of covariational properties, dealing with statistical variation, and fitting parameters with methods like OLS.

In terms of mathematical concepts, we draw on the characteristic covariational properties of those function types, which are introduced in secondary education (linear, exponential, and power functions). These characteristic covariational properties are used in the laboratory to select and compare potential functional models based on assumptions about the framing context (e. g., by making the corresponding covariational behavior plausible in the specific context). We intended to allow students to make informed decisions during structural model selection in specific contexts. The meaning of the parameters in a specific function type is often closely connected to these characteristic properties (e. g., the factor in the exponent as a measure of the strength of the relative change of the dependent value per unit of absolute change in the independent value). This connection is used when model application goes beyond considering specific values of the resulting functions (e. g., at future time points), but addresses questions about the nature of the described relationship. In terms of statistical literacy, this more conceptual approach intends to allow students to engage in in-depth consideration of mathematical models from a structural perspective by connecting the specific mathematical properties with assumptions and conclusions about the corresponding context. First observations indicate that many students struggled to grasp and apply the concept of covariational properties effectively (Mohr & Ufer, 2023). This struggle can be attributed to insecure understanding of both the concept of covariation itself, and in particular of the specific properties of the function types, and its application in selecting and comparing suitable models. It appears that despite the relevance of these properties being introduced in secondary education, students faced difficulties in activating this knowledge or lacked it altogether. Indeed, other properties of the function types may play a stronger role in current teaching practice, indicating a need to reconsider this practice from the perspective of statistical literacy. In our laboratory, this hindered students' ability to critically engage with model selection and comparison. An even stronger preparation of these properties might be necessary to remedy this issue in the short run.

In terms of statistical concepts, the nature of sources of statistical variability (model vs. residual) are central to data-based modeling. This implies that a functional model should approximate, rather than exactly interpolate, the data points (Engel, 2016). This perspective is crucial for balancing the model's fit against its interpretability and the plausibility of its application, a balance essential for developing statistical literacy. Contrary to potential concerns, the engagement of students with statistical variation did not manifest in attempts to interpolate all data points exactly with their models. Given the widespread evidence of students' problems with understanding statistical variation (e.g., Garfield et al., 2008), this can be seen as a positive outcome of engaging with the laboratory, indicating the understanding among students that real-world data cannot be perfectly mapped by mathematical models. This dovetails similar positive effects of Engel et al.'s (2008) approach. Interestingly however, a substantial number of students made efforts to align the function precisely with the first (left-most) point of the dataset. This behavior suggests a partial recognition of the principle that models should serve as approximations rather than exact interpolations of the observed data, reflecting an intermediate misunderstanding or oversimplified view of statistical variation. But it's essential to recognize that understanding statistical variation involves more than avoiding this common pitfall. It requires actively considering the sources of variation and how they impact the model's fit and interpretability. Thus, more work might be needed to address and develop students' understanding of (residual) variation in data-based modeling.

Fitting parameters using methods such as OLS and employing SQS introduce students to the iterative, technical processes involved in model fitting, addressing a more technical side of statistical understanding alongside the corresponding conceptual ideas such as measures of model-data-fit. Observations from the implementation of this approach reveal that students successfully engaged with the presented OLS technique for fitting parameters to a given dataset, but also could describe the role of the SQS from a more conceptual perspective (Mohr & Ufer, 2023). Furthermore, the use of SQS for model comparison emerged as a particularly effective tool. By engaging in this process, students applied a quantitative measure to evaluate the fit of different models to the data. Observations show that students could do this on their own in the transfer phase of the laboratory. The use of digital applets may have played a crucial role in the success. By simplifying these concepts in terms of calculation load, and integrating the measures throughout the whole learning environment, students were able to grasp the essence of model-data-fit in the context of data-based modeling. It should be mentioned that the use of technology without adequate theoretical foundation could lead to students being able to perform certain tasks, but not understanding how to interpret certain results due to a lack of understanding of the mathematical concepts. Students used an applet in the application phase of our student laboratory where the SQS is directly indicated. Considering that only a third of the students use the contextual interpretation of the covariational property for model comparison, but almost all argue based on the SQS, further validation methods need to be introduced, to avoid seeing the SQS as an absolute measure of model quality. The integration of the SQS in the applet might at this point reinforce the focus on it and detract from the application of the mathematical concept of the covariational property of functions and their interpretability in context as a means of model comparison. Finally, we did not discuss the trade-off between the number of parameters and model fit in our laboratory (note that all focused function types had two free parameters), which might be another important subsequent learning step towards a broader coverage of statistical literacy.

6.2 The data-based modeling process framework

We developed a simplified data-based modeling process based on existing frameworks and selected specific aspects from them (Bargagliotti et al., 2020; Lee et al., 2022; Wassner & Proemmel, 2022; Wild & Pfannkuch, 1999). The aim was to elucidate the intricacies of data analysis and modeling, making them accessible and engaging for students. In our data-based modeling process, we described four process phases, that were implemented with age-appropriate activities.

This starts with the selection of a specific function as a model for the context (model selection). The students argue which information or knowledge about the context speaks for or against a specific function type. The relationship in the data is then to be represented by a model function with specific parameter values. Parameter estimation yields parameters to describe the functional relationship numerically (model fitting). Methods to estimate a suitable function may range from informal processes—such as visually fitting a graph to the data—to more sophisticated modeling techniques—such as ordinary least square techniques. This was implemented using applets. These technologies enabled students to explore mathematical models in an interactive and visual manner, which would not have been possible without these tools. Hence, we emphasized the examination and manual determination of the SQS different models can be fitted to data, each based on distinct mathematical properties. Using SQS in terms of model-data fit and the interpretation of the covariational property in context allows to compare different models (model comparison). A clear limitation of our process is that the function obtained through model comparison is not further validated. Thus, our data-based modeling approach could benefit from the introduction of more specific techniques, such as cross-validation, time series validation, or bootstrapping. This could also be implemented through the design and integration of suitable applets. Finally, model application is about applying the model to make statements about the specific problem or context. This includes using the model for prediction and using the model to arrive at context-related decisions, but also making statements about the dynamics of the system based on the covariational properties of the selected function type.

It's worth noting that the described data-based modeling process served as an introduction, laying the groundwork for potential expansion into more sophisticated methods as school students' skills progress. These could encompass advanced techniques such as residual plot analysis, utilization of statistical software, or the validation methods mentioned before, allowing students to gradually explore the complexities of data analysis (Biehler et al., 2011). Moreover, not all process phases of data-based modeling could be implemented in our task. In particular, representing data, specially using digital representations, might be considered as an additional phase in the future (Eichler & Vogel, 2013; Lee et al., 2022). However, prior works have introduced this phase on the lower secondary level (Podworny et al., 2022), so this might not entail specific design challenges when introducing data-based modeling for upper secondary school students.

To conclude, the observations from the implementation of our data-based modeling process indicate that students are capable of understanding and implementing the outlined process. They can follow and learn to actively implement the phases and adopt the techniques, demonstrating their ability to apply the data-based modeling process to the tasks at hand, including the ability to provide conceptual explanations for their statistical work. However, as previously mentioned, there are challenges related to the entailed mathematical conceptual understanding, particularly concerning covariational properties. This issue underscores the need for a deeper foundational grasp of these properties to enhance their application in modeling.

6.3 Choice of contexts

In designing our data-based modeling activities, we aimed to implement an interconnected approach as described by Pfannkuch et al. (2018), which emphasizes learning not only about mathematical and statistical concepts and tools but also about the associated real-world phenomena. To achieve this, we selected contexts where pre-existing theories or concepts about the phenomena do not clearly define the function classes. This choice was intentional to facilitate the interaction between structure-oriented and data-based approaches. While contexts with readily identifiable structures leading to covariational relationships might be more suitable for promoting purely structural modeling—as we attempted to illustrate in the introduction and in Station 1—we recognized that such contexts might conflict with the goal of engaging students through authentic, interesting, and relevant real-world scenarios (Gal, 2022; Wassner & Proemmel, 2022). Therefore, we opted to have students fit different models to the data and compare them, allowing for a deeper exploration of how various models can represent the same phenomenon. The objective was for the fitted curves to help students understand the dynamics of the observed phenomena, thereby contributing to theory building. In Station 3 (Activity 2) students were encouraged to interpret and analyze the developed models with respect to their covariational properties within the given context. This approach aimed to develop their understanding of the dynamics in the corresponding context, embodying the interconnected approach by linking mathematical modeling with meaningful real-world applications. We acknowledge that the three function types we focused on—linear, exponential, and power functions—have inherent weaknesses in structurally capturing the complexities of certain contexts. As students' knowledge progresses, these basic models can and should be replaced or supplemented with more sophisticated ones that offer better fits and more accurate representations of real-world phenomena. Furthermore, to keep the learning curve manageable, we were unable to incorporate certain aspects that are nonetheless central to the data-based modeling process and the promotion of statistical literacy. Specifically, we did not address questions such as how the data were collected, the uncertainties inherent in the data, whether the data are suitable for answering the questions posed within the context, and which alternative data might be better suited for the analysis. While these considerations are crucial for a comprehensive understanding of data-based modeling and for developing statistical literacy, including them would have added complexity that might overwhelm students at an introductory level. Our primary focus was to provide upper secondary school students with an accessible entry point into data-based modeling processes, laying a foundation upon which these more advanced aspects can be built in future learning experiences.

6.4 Design of the implementation within the student laboratory

In the student laboratory for data-based modeling, we implemented several content-related and methodical design principles. These principles intended to collectively contribute to creating a learning environment that not only fosters an understanding of data-based modeling but also promotes important social and cognitive skills such as critical thinking, problem-solving, and teamwork (e.g., Woithe et al., 2022). Structuring the process into manageable learning phases, encouraging collaboration and interaction among students, targeted support through scaffolding, and the integration of modern technologies were all aimed at creating a motivating and effective learning experience. We will exemplarily delve into implementation of the part-task approach and the provision of scaffolding, discussing their application and our first experiences.

The part-task approach (Hasher, 1971) applied in our student laboratory is based on the principle of dividing complex learning tasks into smaller, manageable segments. Current views suggest that part-task approaches can contribute to complexity reduction (e.g., Schukajlow et al., 2015; Zöttl et al., 2010), but that beforehand an overview of the complexity of the entire problem type must be provided (implemented in our case through the introduction), and that opportunities for the combined application of all subprocesses are necessary (implemented through the transfer phase, which is fully processed). In the student laboratory, we addressed this by first offering an introduction to give students a holistic view of the modeling process. We then divided the implementation into four learning phases along our process framework: model selection, model fitting, model comparison, and model application. Each of these learning phases allowed students to engage with specific aspects of the modeling process before carrying out the entire process. Each phase was further divided into several activities. We deliberately chose a linear representation for the part-task approach to achieve the desired complexity reduction from didactic considerations and to make the structural logic of the overall process comprehensible for the learners. This approach aimed to reduce the cognitive load on students, as they needed to concentrate on only one specific aspect of the process at a time (van Merrienboer et al., 2003). Such an approach is often applied in methods oriented towards process models, such as heuristic solution examples (e.g., Schukajlow et al., 2015; Zöttl et al., 2010). However, this part-task method might inhibit the iterative loops and validation processes that are central to modeling. If students encounter a poor fit during the model fitting phase, they may not be able to revisit and reassess the structural considerations made during the model selection phase, since these decisions were provided during the initial learning phase. This separation could prevent students from engaging in the full iterative nature of modeling, where adjusting earlier assumptions based on new findings is essential for refining models. Nonetheless, we included detours and non-linear pathways locally to emphasize that the process is not strictly linear but heuristic. By doing so, we highlight that the modeling process is iterative and dynamic, reflecting how students naturally engage with modeling tasks. The supervising pre-service teachers were instructed to point out these aspects and to emphasize the iterative loops of modeling to the students. Finally, opportunities to engage in the full process were provided in the transfer phase, where all phases were applied collectively. According to our informal observations, reports by tutoring pre-service teachers, and the students' self-assessment, this method, including the choice of specific activities, was well accessible for most students. The positive outcomes (Mohr & Ufer, 2023), particularly in the transfer task, reflect this impression in terms of students' performance on a complete task, indicating that coordinating and iteratively refining the separate parts was not a problem for the participants. The contextual questions and data in the transfer task were, however, structurally quite similar to those in the learning phases. It remains an open question how well students can orchestrate the entire data-based modeling process in future problems that deviate more strongly from this common structure. Broadening the range of tasks and scenarios to better equip students with the competences needed to apply data-based modeling principles across a wider spectrum of contexts might require longer time scales than that of a student laboratory.

The provision of scaffolding was implemented by material-based strategies (e. g., language support, feedback by the online system) complemented by adaptive scaffolding provided by pre-service teachers. Material-based support was directly integrated into the collaboration script that structured student work, offering hints, instructions, and task-specific support tailored to the students' cognitive requirements and capabilities. This approach aimed to challenge students appropriately while providing them the support needed to extend their learning. Additionally, pre-service teachers played an important role in facilitating learning, offering content-related and affective support while promoting a constructive learning environment. This dual approach to support was designed to enable students to work autonomously while having access to necessary assistance. However, support by pre-service teachers will most likely vary in terms of quality, as also indicated by within-group dependencies in learning gain (Mohr & Ufer, 2023). This raises questions about which support characteristics vary between pre-service teachers and influence students' learning in the student laboratory. Drawing on research on tutoring or scaffolding processes (van de Pol et al., 2010) or models of instructional quality might help to understand how different aspects of support affect students' engagement, learning processes, and the development of their data-based modeling skills. On the other hand, a variation of specific design principles is also conceivable. Such studies could provide deeper insights into specific and combined effects of the applied design principles.

7. Outlook

The data-based modeling process and the student laboratory have laid a foundation for enhancing statistical literacy, nevertheless there are several areas where future implementations can be refined and improved. The current process lacks a comprehensive approach to model validation, particularly in verifying the function obtained through model comparison. Incorporating explicit validation phases could enhance the depth of students' statistical analysis skills, but it would most likely also require a treatment of sampling variance. Secondly, there is a need for broadening the range of tasks and scenarios in which the data-based modeling process is applied. The current approach, while effective in introducing students to the fundamentals of data-based modeling, primarily involves tasks with similar structures and excludes central activities such as question or hypothesis generation (Fischer et al., 2014), preparing and implementing data collection, or preparing data for analysis. Lastly, there is a notable gap in students' understanding and application of covariational properties within the modeling process. Enhancing students' understanding of these properties is important for enabling them to select, compare, and apply appropriate models to various contexts more effectively.

Notes

[1] https://cast.itunes.uni-muenchen.de/clips/ASwE33Rv8C/vod/online.html

[2] https://cast.itunes.uni-muenchen.de/clips/Jw5ZlHyXwk/vod/online.html

Acknowledgement

We thank the reviewers for their helpful and constructive remarks and comments.

This project is part of the "Qualitätsoffensive Lehrerbildung", a joint initiative of the Federal Government and the Länder which aims to improve the quality of teacher training. The programme is funded by the Federal Ministry of Education and Research. The authors are responsible for the content of this publication.

References

Bakker, A. (2018). *Design research in education: A practical guide for early career researchers*. Routledge.

Bargagliotti, A., Franklin, C., Arnold, P., Gould, R., Johnson, S., Perez, L., & Spangler, D. A. (2020). *Pre-K–12 guidelines for assessment and instruction in statistics education II (GAISE II): A framework for statistics and data science education*. *American Statistical Association*.

Ben-Zvi, D., & Garfield, J. (2004). Statistical literacy, reasoning, and thinking: Goals, definitions, and challenges. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (pp. 3–15). Springer Netherlands. https://doi.org/10.1007/1-4020-2278-6_1

Biehler, R., Hofmann, T., Maxara, C., & Prömmel, A. (2011). *Daten und Zufall mit Fathom - Unterrichtsideen für die SI und SII mit Software-Einführung*. Schroedel.

Biehler, R., & Schulte, C. (2018). Perspectives for an interdisciplinary data science curriculum at German secondary schools. In R. Biehler, D. Budde, D. Frischemeier, B. Heinemann, S. Podworny, C. Schulte, & T. Wassong (Eds.), *Paderborn symposium on data science education at school level 2017: The collected extended abstracts*, 2–14, Germany: Paderborn.

Blum, W., & Leiß, D. (2007). 5.1 - How do students and teachers deal with modelling problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling* (pp. 222–231). Woodhead Publishing. https://doi.org/10.1533/9780857099419.5.221

Borovcnik, M. (2005). Probabilistic and Statistical Thinking. In M. Bosch (Ed.), *Proceedings of the Fourth Congress of the*

European Society for Research inMathematics Education, 485–506, Spain: Barcelona.

Borromeo Ferri, R. (2007). Von individuellen Modellierungsverläufen zur empirischen Unterscheidung von Phasen im Modellierungsprozess. In *Beiträge zum Mathematikunterricht 2007, 41. Jahrestagung der Gesellschaft für Didaktik der Mathematik*, Germany: Berlin.

Burrill, G. (2014). Tools for learning statistics: Fundamental ideas in statistics and the role of technology. In T. Wassong, D. Frischemeier, P. R. Fischer, R. Hochmuth, & P. Bender (Eds.), *Mit Werkzeugen Mathematik und Stochastik lernen – Using tools for learning mathematics and statistics* (pp. 153–164). Springer Fachmedien Wiesbaden. https://doi.org/10.1007/978-3-658-03104-6_12

Census at School. (2021, 01. January). *Data detectives use PPDAC*. https://new.censusatschool.org.nz/wp-content/uploads/2022/02/data-detective-older-2021.pdf

Chen, J., Wang, M., Kirschner, P. A., & Tsai, C.-C. (2018). The role of collaboration, computer use, learning environments, and supporting strategies in CSCL: A meta-analysis. *Review of Educational Research*, *88*(6), 799–843. https://doi.org/10.3102/0034654318791584

Eichler, A., & Vogel, M. (2013). *Leitidee Daten und Zufall* (2 ed.). Springer Spektrum.

Eid, M., Gollwitzer, M., & Schmitt, M. (2017). *Statistik und Forschungsmethoden* (5 ed.). Beltz Verlag.

Engel, J. (2010). Parameterschätzen in logistischen Wachstumsmodellen. *Stochastik in der Schule*, *30*(1), 13–18.

Engel, J. (2016). Funktionen, Daten und Modelle: Vernetzende Zugänge zu zentralen Themen der (Schul-) Mathematik. *Journal für Mathematik-Didaktik*, *37*, 107-139. https://doi.org/10.1007/s13138-016-0094-4

Engel, J. (2018). *Anwendungsorientierte Mathematik: Von Daten zur Funktion* (2 ed.). Springer Spektrum Berlin.

Engel, J., & Kuntze, S. (2011). From data to functions: Connecting modelling competencies and statistical literacy. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling*, 397–406, Netherlands: Dordrecht.

Engel, J., Sedlmeier, P., & Wörn, C. (2008). Modelling Scatterplot data and the signal to noise metaphor: towards statistical literacy for pre-service teachers. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*,

Euler, M., & Schüttler, T. (2020). Schülerlabore. In E. Kircher, R. Girwidz, & H. E. Fischer (Eds.), *Physikdidaktik - Methoden und Inhalte* (pp. 127–166). Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-662-59496-4_5

Fischer, F., Kollar, I., Ufer, S., Sodian, B., Hussmann, H., Pekrun, R., Neuhaus, B., Dorner, B., Pankofer, S., & Fischer, M. (2014). Scientific reasoning and argumentation: Advancing an interdisciplinary research agenda in education. *Frontline Learning Research*, *2*(3), 28–45. https://doi.org/10.14786/flr.v2i2.96

Fleischer, Y., Biehler, R., & Schulte, C. (2022). Teaching and learning data-driven machine learning with educationally designed Jupyter notebooks. *Statistics Education Research Journal*, *21*(2). https://doi.org/10.52041/serj.v21i2.61

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Reidel.

Gal, I. (2002). Adults' statistical literacy: Meanings, components, responsibilities. *International Statistical Review*, *70*(1), 1–25. https://doi.org/10.1111/j.1751-5823.2002.tb00336.x

Gal, I. (2019). Understanding statistical literacy: About knowledge of contexts and models. In J. M. Contreras, M. M. Gea, M. M. López-Martín, & E. Molina-Portillo (Eds.), *Proceedings of the III International Virtual Congress on Statistical Education*, Online conference.

Gal, I. (2022). Critical understanding of civic statistics: Engaging with important contexts, texts, and opinion suestions. In J. Ridgway (Ed.), *Statistics for empowerment and social engagement: Teaching civic statistics to develop informed citizens* (pp. 323–343). Springer International Publishing. https://doi.org/10.1007/978-3-031-20748-8_13

Gal, I., Nicholson, J., & Ridgway, J. (2022). A conceptual framework for civic statistics and its educational applications. In J. Ridgway (Ed.), *Statistics for empowerment and social engagement: Teaching civic statistics to develop informed citizens* (pp. 37–66). Springer International Publishing. https://doi.org/10.1007/978-3-031-20748-8_3

Garfield, J., & Ben-Zvi, D. (2005). A framework for teaching and assessing reasoning abaout variability. *Statistics Education Research Journal*, *4*(1), 92–99. https://doi.org/10.52041/serj.v4i1.527

Garfield, J. B., Ben-Zvi, D., Chance, B., Medina, E., Roseth, C., & Zieffler, A. (2008). Learning to reason about variability. In J. B. Garfield, D. Ben-Zvi, B. Chance, E. Medina, C. Roseth, & A. Zieffler (Eds.), *Developing Students' Statistical Reasoning: Connecting Research and Teaching Practice* (pp. 201–214). Springer Netherlands. https://doi.org/10.1007/978-1-4020-8383-9_10

Greefrath, G. (2018). *Anwendungen und Modellieren im Mathematikunterricht* (2 ed.). Springer Spektrum Berlin.

Greefrath, G., Hertleif, C., & Siller, H.-S. (2018). Mathematical modelling with digital tools - a quantitative study on mathematising with dynamic geometry software. *ZDM – Mathematics Education*, *50*(1-2), 233–244. https://doi.org/10.1007/s11858-018-0924-6

Hammer, S., & Ufer, S. (2023). Professional competence of mathematics teachers in dealing with tasks in lesson planning. *Teaching and Teacher Education*, *132*, 104246. https://doi.org/10.1016/j.tate.2023.104246

Hasher, L. (1971). Retention of free recall learning: The whole-part problem. *Journal of Experimental Psychology*, *90*, 8–17. https://doi.org/10.1037/h0031350

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research*, *77*(1), 81–112. https://doi.org/10.3102/003465430298487

Kaiser, G., Blum, W., Borromeo Ferri, R., & Greefrath, G. (2015). Anwendungen und Modellieren. In R. Bruder, L. Hefendehl-Hebeker, B. Schmidt-Thieme, & H.-G. Weigand (Eds.), *Handbuch der Mathematikdidaktik* (pp. 357–384). Springer Spektrum.

Kockelkorn, U. (2000). *Lineare statistische Methoden*. Oldenbourg Wissenschaftsverlag. https://doi.org/10.1515/9783486787825

Kollar, I., Wecker, C., & Fischer, F. (2018). Scaffolding and scripting (computer-supported) collaborative learning. In F. Fischer, C. E. Hmelo-Silver, S. R. Goldmann, & P. Reimann (Eds.), *International handbook of the learning sciences: The learning sciences* (pp. 340–350). Routledge. https://doi.org/10.4324/9781315617572

Le, H., Janssen, J., & Wubbels, T. (2018). Collaborative learning practices: teacher and student perceived obstacles to effective student collaboration. *Cambridge Journal of Education*, *48*(1), 103–122. https://doi.org/10.1080/0305764X.2016.1259389

Lee, H. S., Mojica, G. F., Thrasher, E. P., & Baumgartner, P. (2022). Investigating data like a data scientist: Key practices and processes. *Statistics Education Research Journal*, *21*(2), Article 3. https://doi.org/10.52041/serj.v21i2.41

Leiss, D., Ehmke, T., & Heine, L. (2024). Reality-based tasks for competency-based education: The need for an integrated analysis of subject-specific, linguistic, and contextual task features. *Learning and Individual Differences*, *114*, 102518. https://doi.org/10.1016/j.lindif.2024.102518

LernortLabor. (2023). *Schülerlabor-Atlas*. https://www.schuelerlabor-atlas.de

Lewalter, D. (2020). Schülerlaborsbesuche aus motivationaler Sicht unter besonderer Berücksichtigung des Interesses. In K. Sommer (Ed.), *Handbuch Forschen im Schülerlabor: Theoretische Grundlagen, empirische Forschungsmethoden und aktuelle Anwendungsgebiete* (pp. 63–70). Waxmann.

Lichti, M. (2019). *Funktionales Denken fördern*. Springer Spektrum.

Mandl, H., & Friedrich, H. F. (2006). *Handbuch Lernstrategien*. Hogrefe Verlag.

Mohr, M., & Ufer, S. (2023). Measuring data-based modeling skills in a collaborative setting. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel, & M. Tabach (Eds.). *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education, Vol. 3*, 355–362, Israel: Haifa.

Molnár, G., & Csapó, B. (2018). The efficacy and development of students' problem-solving strategies during compulsory schooling: Logfile analyses. *Frontiers in Psychology*, *9*. https://doi.org/10.3389/fpsyg.2018.00302

Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. In S. Papastavridis & A. Gagatsis (Eds.). *3rd Mediterranean Conference on Mathematical Education*, 116–124, Greece: Athens.

Niss, M., Blum, W., & Galbraith, P. (2007). Introduction. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 3–32). Springer US. https://doi.org/10.1007/978-0-387-29822-1_1

Nocedal, J., & Wright, S. J. (2006). *Numerical Optimization* (2 ed.). Springer New York. https://doi.org/10.1007/978-0-387-40065-5

Ostkirchen, F., & Greefrath, G. (2022). Case study on students' mathematical modelling processes considering the achievement level. *Modelling in Science Education and Learning*, *15*(1), 137–150. https://doi.org/10.4995/msel.2022.16506

Pfannkuch, M., Ben-Zvi, D., & Budgett, S. (2018). Innovations in statistical modeling to connect data, chance and context. *ZDM – Mathematics Education*, *50*(7), 1113–1123. https://doi.org/10.1007/s11858-018-0989-2

Podworny, S., Hüsing, S., & Schulte, C. (2022). A Place for a Data Science Project in School: Between Statistics and Epistemic Programming. *Statistics Education Research Journal*, *21*(2), Article 6. https://doi.org/10.52041/serj.v21i2.46

Pohlkamp, S. (2022). „Daten und Zufall“ mit „Modellieren“ verknüpfen: Bildungspotenzial am Beispiel von Datendarstellungen zum Arktis-Eis. *Stochastik in der Schule*, *42*(2), 22–31.

Ralle, B. (2020). Empirische Forschung in Schülerlaboren – Eine anspruchsvolle Aufgabe. In K. Sommer, J. Wirth, & M. Vanderbeke (Eds.), *Handbuch Forschen im Schülerlabor: theoretische Grundlagen, empirische Forschungsmethoden und aktuelle Anwendungsgebiete* (pp. 13–20). Waxmann.

Reinhold, F. (2019). *Wirksamkeit von Tablet-PCs bei der Entwicklung des Bruchzahlbegriffs aus mathematikdidaktischer und psychologischer Perspektive*. Springer Spektrum Wiesbaden. https://doi.org/10.1007/978-3-658-23924-4

Ryan, R. M., & Deci, E. L. (2020). Intrinsic and extrinsic motivation from a self-determination theory perspective: Definitions, theory, practices, and future directions. *Contemporary Educational Psychology*, *61*, 101860. https://doi.org/10.1016/j.cedpsych.2020.101860

Schukajlow, S., Kolter, J., & Blum, W. (2015). Scaffolding mathematical modelling with a solution plan. *ZDM – Mathematics Education*, *47*(7), 1241–1254. https://doi.org/10.1007/s11858-015-0707-2

Sproesser, U., & Brühne, T. (2022). Umwelterschließung meets Datenauswertung – In der Auseinandersetzung mit Klimadaten den Nutzen statistischer Kennwerte und Diagramme erkennen. *Stochastik in der Schule*, *42*(3), 12–21.

Tschirgi, J. E. (1980). Sensible reasoning: A hypothesis about hypotheses. *Child Development*, *51*(1), 1–10. https://doi.org/10.2307/1129583

van de Pol, J., Volman, M., & Beishuizen, J. (2010). Scaffolding in teacher–student interaction: A decade of research. *Educational Psychology Review*, *22*(3), 271–296. https://doi.org/10.1007/s10648-010-9127-6

van Merrienboer, J. J. G., Kirschner, P. A., & Kester, L. (2003). Taking the load off a learner's mind: Instructional design for complex learning. *Educational Psychologist*, *38*(1), 5–13. https://doi.org/10.1207/S15326985EP3801_2

Vogel, F., Wecker, C., Kollar, I., & Fischer, F. (2017). Socio-cognitive scaffolding with computer-supported collaboration scripts: A meta-analysis. *Educational Psychology Review*, *29*(3), 477–511. https://doi.org/10.1007/s10648-016-9361-7

Vogel, M. (2014). Visualisieren – Explorieren – Strukturieren: Multimediale Unterstützung beim Modellieren von Daten durch Funktionen. In T. Wassong, D. Frischemeier, P. R. Fischer, R. Hochmuth, & P. Bender (Eds.), *Mit Werkzeugen Mathematik und Stochastik lernen – Using tools for learning mathematics and statistics* (pp. 97–111). Springer Fachmedien, Wiesbaden. https://doi.org/10.1007/978-3-658-03104-6_8

Vogel, M., & Eichler, A. (2014). Von Pinguinen, Wasser, Daten und Funktionen – Phänomene und ihre Modellierungen im Mathematikunterricht. In U. Sproesser, S. Wessolowski, & C. Wörn (Eds.), *Daten, Zufall und der Rest der Welt* (pp. 267–280). Springer Spektrum.

Wassner, C., & Proemmel, A. (2022). Civic statistics at school: Reasoning with real data in the classroom. In J. Ridgway (Ed.), *Statistics for empowerment and social engagement: Teaching civic statistics to develop informed citizens* (pp. 417–444). Springer International Publishing. https://doi.org/10.1007/978-3-031-20748-8_17

Weinert, F. E. (2001). Concept of competence: A conceptual clarification. In D. S. Rychen & L. H. Salganik (Eds.), *Defining and selecting key competencies* (pp. 45–65). Hogrefe & Huber Publishers.

Wickens, C. D., Hutchins, S., Carolan, T., & Cumming, J. (2013). Effectiveness of part-task training and increasing-difficulty training strategies: A meta-analysis approach. *Human Factors*, *55*(2), 461–470. https://doi.org/10.1177/0018720812451994

Wild, C. J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review*, *67*(3), 223–248.

Woithe, J., Müller, A., Schmeling, S., & Kuhn, J. (2022). Motivational outcomes of the science outreach lab S'Cool LAB at CERN: A multilevel analysis. *Journal of Research in Science Teaching*, *59*(6), 930–968. https://doi.org/10.1002/tea.21748

Wood, D., Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry*, *17*(2), 89–100. https://doi.org/10.1111/j.1469-7610.1976.tb00381.x

Zöttl, L., Ufer, S., & Reiss, K. (2010). Modelling with Heuristic Worked Examples in the KOMMA Learning Environment. *Journal für Mathematik-Didaktik*, *31*(1), 143-165. https://doi.org/10.1007/s13138-010-0008-9

Zöttl, L., Ufer, S., & Reiss, K. (2011). Assessing modelling competencies using a multidimensional IRT approach. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.). *Trends in Teaching and Learning of Mathematical Modelling*, 427–437, Netherlands: Dordrecht.

Anschrift der Verfasser:innen

Matthias Mohr
Ludwig-Maximilians-Universität München
Mathematisches Institut Didaktik der Mathematik
Theresienstr. 39
80333 München
mohr@math.lmu.de

Luzia Hofer
Ludwig-Maximilians-Universität München
Mathematisches Institut Didaktik der Mathematik
Theresienstr. 39
80333 München
lhofer@math.lmu.de

Stefan Ufer
Ludwig-Maximilians-Universität München
Mathematisches Institut Didaktik der Mathematik
Theresienstr. 39
80333 München
ufer@math.lmu.de