Development of Early Spatial Perspective-Taking – Toward a Three-Level Model

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Zusammenfassung: *Ausgehend von einen Zwei-Level-Modell der Entwicklung räumlichen Perspektivübernahme führten inkonsistente Befunde zu Ausdifferenzierungen in Drei-Level-Modellen. Dieser Beitrag entwickelt und validiert ein mögliches integriertes Drei-Level-Modell durch zwei Zugänge. Forschungsergebnisse werden unter Verwendung des Drei-Level-Modells neu interpretiert und Daten einer Interviewstudie mit 95 Erstklässler*innen reanalysiert. Erst die Begründungen der Kinder ließen erkennen, dass Level-3-Aufgaben schwieriger waren als Level-2-Aufgaben. Unsere Ergebnisse zeigen, dass ein Drei-Level-Modell sich als passfähig zur Beschreibung der Entwicklung der räumlichen Perspektivübernahme erweist.*

Abstract: *The development of spatial perspective-taking has been described in a two-level model. Inconsistent research results led to more differentiated three-level models. This study draws upon these ideas and aims at developing and validating an integrated three-level model by choosing two approaches. First, prior research results will be interpreted by using the three-level model. Second, we re-analyze data from an interview study with 95 first graders. In particular, the children's explanations for their solutions showed that level-3-tasks were more difficult for them than level-2-tasks.Our findings suggest that a three-level model is appropriate to describe the development of spatial perspective-taking more precisely.*

1. Introduction

Spatial reasoning, a focus of geometry education in primary school, has received much interest in recent years. This can be seen, for example, in the ZDM special issue "Geometry in the Primary School" (2015). The editors of this issue, Sinclair and Bruce (2015), and Mulligan (2015)'s commentary to it emphasize the theoretical and empirical interest in geometrical learning of children between pre-school and lower secondary school.

One explanation for the interest in geometrical learning in early childhood might be that there is a growing body of literature providing evidence for a relationship between spatial and mathematical abilities. Several studies found significant correlations between measures of both kinds of abilities (e.g., Graß & Krammer, 2018; Verdine et al., 2014). Moreover, Cheng and Mix (2014) showed that training of spatial abilities also improves mathematical abilities, so that a causal relationship can be assumed. Wai, Lubinski, and Benbow (2009) provided evidence that spatial abilities are not only crucial for learning mathematics, but more generally for entering STEM fields successfully.

Spatial abilities are modeled best as multidimensional cognitive abilities, and spatial perspective-taking can be conceptualized as one component of spatial abilities (e.g., Hegarty & Waller, 2005). It is defined as a set of cognitive abilities that allow individuals to imagine how objects appear from another point of view than one's own (Cox, 1977a). To solve spatial perspective-taking tasks successfully, individuals draw upon knowledge about the relationship between views and positions. For example, two individuals do not see an object in the same way due to their different perspectives. For many tasks, the application of this kind of knowledge is, however, not sufficient. These tasks require a profound understanding of space and how spatial relations might change whenever one mentally moves to another viewer's position.

Empirical studies have addressed spatial perspective-taking abilities of children and adults in different ways: Many studies focused primarily on different tasks that were mastered by different age groups and that were interpreted as stages of development in spatial perspective-taking (e.g., Flavell, Everett, Croft, & Flavell, 1981, van den Heuvel-Panhuizen, Elia, & Robitzsch, 2015). Another focus was on different task characteristics that influence the difficulty of tasks (see Newcombe, 1989, for an overview). These studies have in common that they inferred spatial perspective-taking abilities either from solution rates, that is, from a psychometric point of view, or from the types of errors in a set of predefined tasks, that is, from a cognitive psychological point of view. To gain an overall insight into the children's spatial thinking in perspective-taking tasks, however, it would be fruitful to combine both perspectives as Linn and Petersen (1985) have demonstrated before.

Niedermeyer (2015) addressed this gap in the literature, providing results about the spatial perspective-taking abilities of six-year-old children by asking them to solve tasks and to justify their solutions in an interview. By categorizing the children's errors and explanations, she described the difficulty of different tasks more precisely than it would have been possible by analyzing solution rates or errors alone. The current study re-analyzes empirical data from Niedermeyer (2015) to propose a three-level-model of perspective taking using solution rates, error types, and explanation categories.

2 Three Levels of Spatial Perspective-Taking

Previous studies have proposed that there are different levels of perspective-taking abilities, that is, different kinds of tasks that are mastered at different stages of development. In this section, we first review the classical one-level model of Piaget and Inhelder (1971) and the two-level model of Flavel et al. (1981). We emphasize that, whereas there is general acceptance in the literature about level 1 of Flavel et al. (1981), tasks of level 2 have been a matter of debate. In consequence, different studies have proposed a differentiation of level 2, supposing three different levels of development (Coie, Constanzo, Farnill, 1973; Eisner, 1976; Rosser et al., 1985). We present three different models here and analyze the underlying cognitive processes of the proposed levels in each of the models. Furthermore, we show that the levels of the three different models can be subsumed into an integrated three-level model of perspective-taking abilities.

2.1 One- and Two-Level Models as a Starting Point

One of the first studies that addressed spatial perspective-taking of children was the "three-mountains-task" by Piaget and Inhelder (1971, French first edition 1948). They investigated the abilities of four- to twelve-year-old children in three different tasks requiring spatial perspective-taking. Their results indicated that children until an age of 7 solved the tasks as if every person in another position perceives the mountains in the same way as they do, a behavior that was denoted as egocentrism. Between the age of 7 and 9, the examined children showed awareness that another person has another view but they could not completely detach themselves from their own perspective. Only from the age of 9 onwards, the children were able to solve the three-mountains-task correctly.

In the following years, many other studies showed that children younger than 9 years are not as egocentric as Piaget and Inhelder concluded. They could indeed solve spatial perspective-taking tasks that require to figure out if another observer can see an object or not (e.g., Flavell et al., 1981, Masangkay et al., 1974, van den Heuvel-Panhuizen et al., 2015). Results of these studies were summarized in models that describe perspective-taking abilities at different levels in more detail than the originally proposed egocentric stage of children of this age by Piaget and Inhelder (1971).

Flavell et al. (1981) developed a two-level model of spatial perspective-taking. According to them, level-1-tasks focus on the visibility of objects whereas level-2-tasks include their appearance (see van den Heuvel-Panhuizen et al., 2015). Tasks on level 1 are tasks that can be solved by answering the question "What can be seen?" Masangkay et al. (1974), for example, placed a card vertically between the child and an adult experimenter that showed different pictures on both sides. The child had to conclude which picture is seen by the experimenter. These level-1-tasks were solved by three-year-old children. The corresponding task for level 2 in the study of Masangkay et al. (1974) required to answer the question if the experimenter sees a turtle on a card upside down or right side up, while the card was laying on the table between the child and the experimenter. Level-2-tasks require answering the question: "How is the object seen?"

Different studies confirmed the result that level-1-tasks can be solved by children at the age of 3 to 4: Hughes and Donaldson (1979), and Hobson (1980), for example, used tasks in which the children had to place toy figurines and walls so that one figurine cannot be seen by the other figurines. Most of the children at the age of 3 to 4 could solve these tasks without errors.

For tasks at level 2 that require to coordinate relations and to figure out mentally how the view of an observer at another position looks like, studies showed inconsistent results concerning the age at which they can be mastered.

2.2 Different Three-Level Models

The inconsistent results concerning level-2-tasks led to different suggestions of further differentiations of the two-level model of Flavell et al. (1981). Three studies will be reported in some more detail.

In the first study, Rosser, Ensing, Mazzeo, and Horan (1985) assumed that the inconsistency in the results may be due to different types of tasks and their demands. They concluded that these different task types seem to influence item difficulty and result in experimental conditions that cannot be compared. Furthermore, the authors pointed out that the description of level 2 by Flavell et al. (1981) did not describe sufficiently the demands of different level-2-tasks. Based on theoretical analyses of possible cognitive processes underlying the solving perspective-taking tasks, Rosser et al. (1985) proposed a further elaboration of Flavell et al.'s (1981) level 2.

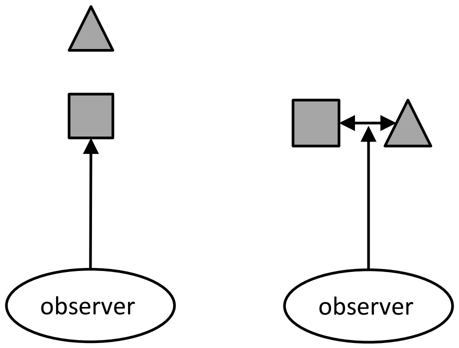

Fig. 1: Different demands of level-2-tasks (on the left; external observer-object-relationship) and level-3-tasks (on the right; internal relations of the object arrangement)

According to Rosser et al. (1985), lower level-2-tasks demand for the realization of a so-called external relationship between the other observer and the object or object arrangement by referring to the part of the object (arrangement) that is in front of the other observer and therefore seen best. Higher level-3-tasks, however, demand for the coordination of internal relations of the object (arrangement) and therefore for an exact inference of how the objects are seen by the other observer (see Rosser et al., 1985). Figure 1 illustrates these different requirements.

As a second approach presented here, Coie, Costanzo, and Farnill (1973), analyzed different types of errors that second, third, and fourth graders made in perspective-taking tasks. Errors were seen as indicators that the child cannot master how objects are seen. A typical example was the choice of pictures that showed the wrong side of a house (the narrow instead of the wide wall). The authors summarized their results as a development "from what is seen (the visibility of objects), to how objects are seen (the appearance of objects), and then to where objects are seen (on the right or left side of the visual field)" (Coie, Costanzo, & Farnill, 1973, p. 176).

A third approach was chosen by Eiser (1976). She separated child and interviewer by a screen and gave both persons the same arrangement of three objects on a rotatable plate. The interviewer rotated his/her arrangement and the child had the task to ask yes-no-questions to find out how the interviewer rotated his/her plate and to rotate his/her own plate in the same way. Eiser (1976) recorded the difference in degrees between the two rotations and the number of questions the child asked until he/she was sure that his/her own plate was in the same orientation as the plate of the interviewer. These measures were combined to an efficiency value. In addition, Eiser (1976) registered the type of the children's questions. She differentiated between "orientation"-questions, which refer to particular sides of the objects or specific features ("Do you see the side of the mouse?", "Can you see the eyes of the snake?"), and "position"-questions that refer to the relation between objects ("Is the mushroom on the left side?" or "Is the mouse behind the mushroom?").

It turned out that children with a high efficiency value (in relation to children of the same age group) often asked "position"-questions whereas "ineffective" children often used "orientation"-questions that only refer to "which" part of the objects is seen (see Eiser, 1976, p. 207).

2.3 Toward an Integrated Model

All perspective-taking tasks require an individual to mentally take another observer's position. Specifying the demands of this mental change, the results of the studies outlined above can be summarized and merged to an integrated model as follows (Figure 2).

Level 3	HOW are the objects seen? (appearance of the objects) COORDINATION OF RELATIONS	Relationships – especially the left-right-relation – between (parts of the) objects (appearance of the objects) INTERNAL RELATIONS
Level 2		Tasks can be solved by answering the question "which part of the object(s) can be seen best/at the front?" (visibility of key features) EXTERNAL RELATIONS
Level 1	WHAT can be seen / cannot be seen? (visibility of objects) PERCEPTION OF OBJECTS	

Fig. 2: An integrated 3-level model of perspective-taking abilities

There is a clear distinction between *what* and *how* as suggested by Flavell et al. (1981). Tasks on level 1 ask for existing features. These questions about the visibility of objects can be mastered more or less in a static manner by the perception of objects. It's a question about a visual feature and not so much about a spatial relation.

Tasks that deal with the appearance of objects, however, are likely to be differentiated regarding the kind of relations that need to be coordinated. Tasks of level 2 focus on the front-back-relation of the object (arrangement). Although a relation inherent in the situation is focused, the question can be reinterpreted into an external observer-object-relation. This reinterpretation allows for a much easier “static” view to key features of the object. Though these have to be key features visible from the observer’s point of view, especially eye-catching features in the front can easily be identified.

In contrast, level-3-tasks ask for relations inherent in the object (arrangement) that cannot be reinterpreted as questions about static key features. Solving these tasks, it is necessary to manage internal relations of the object arrangement thus engaging in a cognitive coordination of relations on a higher level.

Taken together, the previously described studies (Coie et al., 1973; Eiser, 1976; Rosser et al., 1985) can be *theoretically* integrated into a coherent three-level model of perspective taking concentrating on different relations that have to be taken into account. Nevertheless, this model needs further validation.

3. Validation of the Three-Level Model of Spatial Perspective-Taking

We used two approaches to present further arguments for a three-level model of perspective-taking abilities. First, taking a theoretical perspective, we re-analyzed task characteristics in the literature and their difficulty with respect to different relations that have to be taken into account and that were assumed to be mastered at different levels of development.

Second, taking an empirical perspective, we re-analyzed the empirical data from the study of Niedermeyer (2015), thereby testing whether a proposed three-level model can explain the results of this study with 95 first graders.

3.1 Theoretical Approach: Analysis of Task Characteristics that Influence Levels of Perspective-Taking Abilities

A first approach for validating the three-level model for spatial perspective-taking abilities was to review the literature for results giving indications that the three levels describe the development of spatial perspective-taking and the corresponding demands of different tasks appropriately. The following section summarizes results for different task characteristics that were identified as influencing children’s perspective-taking performance and the difficulty of tasks.

3.1.1 Types of tasks

Especially the types of tasks that are used for investigating children’s and adults’ perspective-taking vary a lot between different studies. Most of the results concerning the comparison of different tasks can be interpreted using the level-2/level-3-distinction.

Laurendeau and Pinard (1970), for example, used an object arrangement that is very similar to that used by Piaget and Inhelder (1971): They arranged three differently colored cones with different heights. They compared a *picture-selection-task* with a *position-identification-task* in which the child had to identify the position of another observer from which he/she can see the cones as shown on a picture. The picture-selection-task offered nine photographs also including two photos that did not fit the cones arrangement.

The results showed that the picture-selection-task was much more difficult for children at the age of 4 to 12 than the position-identification-task. Laurendeau and Pinard (1970) traced back this result to the fact that with picture-selection-tasks the children had to consider more than one relation because wrong photographs correspond with the right photograph in some relations. So, relations between the arrangements as internal relations have to be addressed. Therefore, these tasks can be seen as level-3-tasks. The position-identification-task in contrast can be solved by considering only one external relation (normally the front-back-relation). The child regards which cone is seen at the front and positions the other observer near this cone. So, these tasks can be interpreted as level-2-tasks. The lowest solution rates could be found for a special position-identification-task. In this task the child was asked to find the position of the other observer for a picture that didn’t show the cones in the right relationship – without prewarning that a picture can also not fit. This task requires the coordination of front-back- and left-right-relations by arguing “when the red cone is at the front, the blue one must be seen on the right side and not on the left side”.

In this study of Laurendeau and Pinard (1970), picture-selection-tasks can be interpreted as level-3-tasks because the use of wrong photographs requires the coordination of multiple relations. Newcombe (1989) concluded that picture-selection tasks with only right photographs can be solved by considering which object is seen at the front. So, without using wrong photographs, picture-selection-tasks can always be classified as level-2-tasks.

In a similar way it can be explained why *reconstruction-tasks* – the child has to reconstruct the other observer’s view by using symbols or models of several objects – are more difficult than *model-rotation-tasks* – the child has to rotate a model of the object arrangement so that he/she can see the model in the same way as the other observer sees the real arrangement (Rosser et al., 1985). Model-rotation-tasks conserve the intrinsic relations. The only demand is to keep an eye on the extrinsic relation and to figure out which object is seen best in the view of the other observer. So, they can be seen as level-2-tasks. Reconstruction-tasks in contrast require the replication of the other observer's view by arranging single objects. It is not enough to place the right object at the front as the internal relations between different objects must be reconstructed in the right way. If the objects themselves have different sides, the child also has to consider their orientation. Reconstruction-tasks can therefore be classified as level-3-tasks.

For *item- and position-questions* (Which object can be seen on the left side?/Where is object x seen?), the assignment to one of the levels depends a lot on the exact procedure of the task: If only questions about front or back (or objects at the front or back respectively) are asked, these tasks focus on external relations and can be classified as level-2-tasks. If questions about left and right or objects at the left or right side in the other observer's view are asked, these tasks require the coordination of internal relations and are therefore tasks at level 3. But if the coordination of left and right is simplified by using colored stickers on the hands of the child and the hands of the other observer (see Huttenlocher & Presson, 1979), the task can be seen as level-2-task because it can be solved by considering which object is near to a specific hand.

3.1.2 Position of the other observer

There are also inconsistent results in the literature concerning the difficulty of different positions of the other observer. Some studies revealed that tasks are easier to solve when the other observer has a position opposite to the child (Eiser, 1974; Gzesh & Surber, 1985; Presson, 1982). In other studies, tasks with this position of the other observer were solved less often than tasks where the other observer was positioned to the left or to the right of the child (Cox, 1977b; Jacobsen & Waters, 1985; Nigl & Fishbein, 1974; Wraga, Creem, & Proffitt, 2000). All the studies will not be re-analyzed in detail here, because Cox (1977b) already concluded that in studies of the first group the views differed concerning their demands: for example, at the opposite position one object was hidden behind another one. If such differences between views occur with an object arrangement, the tasks must be assigned to different levels of spatial perspective-taking: If a view can be assigned to a position by referring to what is seen at the front, these tasks are level-2-tasks. If the coordination of left and right is necessary, the tasks correspond to level 3 and if tasks can be solved even by considering if an object is seen or not, they must be seen as level-1-tasks.

3.1.3 Type of view

The effect of the other observer's position on the classification of tasks into a level is influenced by the other observer's type of view as could be seen in the previous section.

With objects that are intrinsically oriented (which means that labels as "front", "back", and "side" can be assigned unambiguously), these different sides can affect the task difficulty. Ives and Rakow (1983) for example found out that tasks with a rubber duck whose left and right side must be distinguished are solved more often and with less errors when the rubber duck's sides are marked with differently colored stickers. Without stickers, the left and the right side of the duck can only be distinguished by considering the internal relations of the left-right-orientation (level 3). The stickers provide information about "what" is seen (the red or the blue sticker) and with this preparation the tasks allow for the reference to the external relations and can be interpreted as level-2-tasks. So, if one considers the differences in the levels, the result that tasks with marked ducks are solved better than tasks without stickers is therefore not surprising.

In a study with a dollhouse, Walker and Gollin (1977) found differences between views that show a front section of the house and views that show a corner view. The frontal views were more often assigned correctly than the corner views. This result can also be interpreted in the level-2/level-3-distinction: The frontal views can be solved by considering one specific feature that identifies the specific wall of the house (a window, a chimney, …). With corner views, the features of two walls can be seen by the other observer and must be identified in the range of possible pictures. The analysis of errors in the study showed that many children chose a picture that showed one feature of the two visible walls. They obviously concentrated on "what is seen" and did not manage to consider all visible objects in their intrinsic relations.

Our analyses showed that all these results of different studies about task characteristics that influence the difficulty of perspective-taking tasks can be interpreted reasonably by using the level-2/level-3-distinction as proposed by Rosser et al. (1985).

3.2 Empirical Approach: Interview Study with Six-Year-Old Children

The following section describes the Niedermeyer (2015) interview study with first graders in the context of a possible three-level model of perspective-taking abilities. It also presents the results of a re-analysis of the data, showing that a three-level model might explain different perspective-taking abilities.

The study originally addressed the influence of symmetry in perspective-taking tasks, relying on a set of systematically varied tasks that were designed for school beginners. Students of grade 1 solved the tasks in individual interviews and were asked to explain their solutions. These explanations were classified according to the relevant spatial features and relations that were taken into account.

The study concentrated on level 2 and level 3 of the model by Rosser et al. (1985) because the distinction between level 1 and level 2 (as described in the model of Flavell et al., 1981) has already been established in the literature (see, e.g., van den Heuvel Panhuizen et al., 2015). Several studies have demonstrated that most children have developed level-1 perspective-taking abilities by the age of 3. Level-2 abilities (as described by Flavell et al., 1981 and denoted as level-3 by Rosser et al., 1985) seem to develop starting at the age of 4 to 5 and improve remarkably in the following years (see Frick, Möhring, & Newcombe, 2014, for literature). Therefore, tasks targeting at least level-2 abilities seemed suitable for first-graders.

This study re-analyzed the empirical data from Niedermeyer (2015) to answer the following research question:

> *Does a model involving level 2 and level 3 sufficiently reflect early primary students' spatial perspective-taking abilities?*

The research question was addressed under three different empirical perspectives. First, we took a psychometric perspective, thus comparing solution rates of different groups of tasks that required children to rely on perspective-taking abilities at different levels. Second, we refined our findings by modeling the empirical data using a cognitive perspective, thus quantifying and comparing different types of errors in each of the task groups. Third, we collected further evidence to support our previous findings by taking another cognitive perspective by comparing the different explanations presented for each of the task groups.

3.2.1 Sample and Methods

The sample. 95 first-graders (average age: 6 years 8 months) of two different schools in Germany participated in this study. Table 1 presents the composition of the sample according to classrooms and gender.

	school 1 class A	school 1 class B	school 1 class C	school 2 class D	total
girls	11	10	14	10	45
boys	11	13	15	11	50
total	22	23	29	21	95

Tab. 1: Composition of the sample

The interviews took place in a separate room during lessons in the second to fifth week of the school year, lasted about 15 to 25 minutes, and were videotaped. All interviews were conducted by the same person in order to ensure consistent behaviour.

The tasks. The study relied on picture-selection-tasks because they enable the systematic variation of different aspects and are easy to explain to young children. For all tasks, a square base (40cm×40cm) with four differently coloured toy figurines placed in the centre of each edge was used. In the middle of this base, the interviewer placed 16 different objects one after another. Four photographs of the object

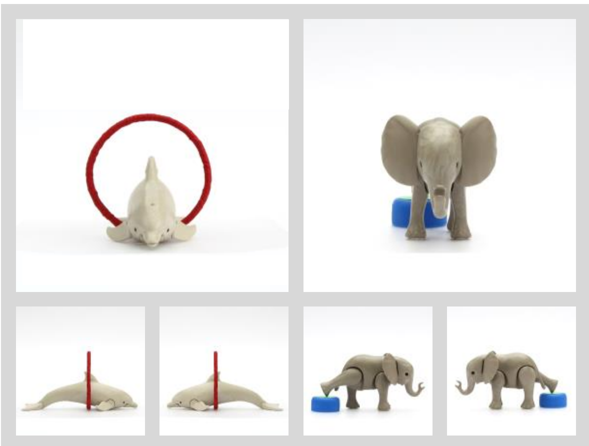

Fig. 3: Examples for symmetric and asymmetric animals and their side-views

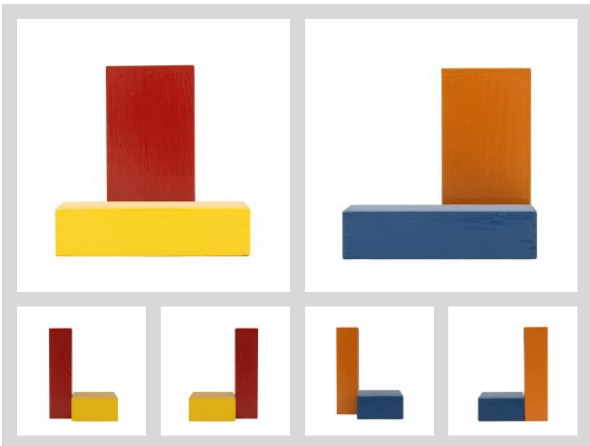

Fig. 4: Examples for symmetric and asymmetric cuboid arrangements and their side views

(depicting the object's four different sides) were positioned between the child and the base. For every task, the child was asked which of the pictures corresponds to the view of one of the toy figurines, saying: "Which of the photographs did the green man take?" When the children gave their answers, they were invited to explain their decisions. Afterwards, they were asked about another toy figurine's view. In every situation, only two views were tested to limit subsequent errors that are rooted in a previous error.

Two different types of objects were used for the study: toy animals that are well-known to children and have clearly determined sides (front, back, left side, right side) and arrangements of two differently coloured cuboids of the same size as abstract objects without distinguishable front, back and sides.

Another aspect that has been varied systematically in the set of tasks was the symmetry of objects. We used objects with and without a vertical symmetry plane. For each object type (animals and cuboid buildings), four symmetric and four asymmetric objects were used. The natural symmetry of the animals was abolished by lifting one leg and adding an item from a circus context.

The symmetric animals were also equipped with items from the circus context to minimize the differences between symmetric and asymmetric animals (see figure 3).

For cuboid tasks, first a symmetric arrangement was built and then an asymmetric arrangement was created by sliding one cuboid orthogonally to the mirror plane (see figure 4).

The variation of symmetry offers the possibility to test tasks of level 2 and 3 with the same objects. Symmetry is a characteristic of objects that leads to views that are symmetric to each other as can be seen in figures 3 and 4. The two side-views of the dolphin and the left cuboid building only differ in their left-right-orientation. There is no other feature that can be considered to distinguish them. The two side-views of the asymmetric objects in contrast can be discriminated by considering the differences in the front-back-relation: on one picture, a feature of the object (the risen leg of the elephant or the orange block) is seen at the front whereas on the other picture it is seen at the back. This characteristic of symmetric and asymmetric objects in spatial perspective-taking tasks allows us to use similar objects for level-2- and level-3-tasks. Other possible tasks for level 2 and level 3 differ more from one another and could therefore be perceived as different tasks more directly.

Apart from symmetry and type of object, two other factors were varied: the object's orientation (parallel or orthogonal to the child's line of sight) and the type of view asked for (side-view or front/back view).

To ensure comparability, every symmetric object was paired with an asymmetric object of the same object type and within such a pair of tasks all other variables were kept constant: the orientation, the arrangement of the pictures, and the two toy figurines whose views should be figured out.

	symmetric objects	asymmetric objects
front- and back-views	level 2	
side-views	level 3	level 2

Tab. 2: Theoretical classification of tasks

Referring to the differences between level 2 and level 3 as described by Rosser et al. (1985), the tasks can be assigned to the levels as follows (see table 2):

Level-3-tasks are

- items in which a *side-view of symmetric objects* must be assigned. They can only be solved by referring to internal relations especially considering the left-right-relation ("the dolphin is looking to the left", "the yellow block can be seen to the left of the red one").

Level-2-tasks are:

- items in which a *side-view of asymmetric objects* must be assigned. They can be solved by considering the difference between the two side-views in the front-back-relation and therefore by relying on the external relation ("he sees the risen leg in the front", "the blue block is sticking out to the front").
- items in which a *front- or back-view of objects* must be assigned. They can also be solved by referring to the external relation by considering the front-back-relation ("the yellow block is in front of the red one") or by considering what is seen from the other position ("there you can see the head of the elephant").

Procedure of analysis. The interview data was analyzed in two ways. The children's decisions were classified with respect to the type of error they made, and the explanations were transcribed and categorized using qualitative content analysis (Mayring, 2010).

The children's answers were classified as follows:

- Correct answer: The child chose the picture that showed the toy figurine's view.
- Egocentric error: The child chose the picture that showed his/her own view instead of that of the toy figurine.
- Inversion error: Being asked about a side-view the child chose the wrong one.
- Ambiguous error: If the child was asked about the view opposite to his/her own of an orthogonally aligned object (so it was a question about a side-view), he/she chose the picture that shows his/her own view. This error could either be classified as an egocentric error or as an inversion error. Therefore, this error was named "ambiguous error".
- Other: all other errors.

The children's explanations were transcribed, including their gestures. In a second step, these explanations were sorted by likeness. The analysis of difficulties and similarities in both situational contexts – animals and cuboids – led to the following category system (see Niedermeyer & Ruwisch, 2014, for more details).

Categories for children's statements.

Category 1. The first category includes all statements that refer to what is seen (at the front). Statements for front- and back-views of animals are for example "he sees the tail", "the head is at the front" and for front- and back-views of cuboid buildings "the blue one is showing to him" or "the orange block is at the front and the blue one at the back". Statements of this category for side-views of symmetric animals do not differentiate exactly between the two side-views ("he sees the belly") because they fit to both side-views. For side-views of asymmetric animals, statements of this category can unambiguously justify the solution by referring to the asymmetric feature at the front or the back of the view ("the risen leg is at his side"). A statement for tasks with asymmetric cuboid buildings is for example "the blue one is more to the front on this picture than on that picture (points to the other side-view)".

Category 2. The second category differs between animal and cuboid tasks because animals have an intrinsic orientation (there is a clear definition where the front is) whereas cuboid buildings don't have such an intrinsic alignment. For the animals, statements of this category refer to their intrinsic orientation by stating, which side can be seen by the other observer ("you can see the elephant from the front", "the figure is looking to his side"). For the cuboid buildings, statements of this category refer to the different sides of the cuboids that can be seen by the other observer. This can be expressed explicitly ("he sees the narrow side of the blue block") or implicitly ("he can see only a little bit of green and more red").

The relation terms "front", "back" and so on can refer to different reference systems. "Behind" can refer to the child's point of view (for example when the child is talking about the blue figurine opposite to him-/herself: "he is standing behind the animal"), to the other observer's point of view ("the blue block is behind the red one"), or concerning the animals to their intrinsic orientation (in this case, "he is standing behind the elephant" refers to the figurine at the tail of the animal). In most cases, only the interpretation as reference to the animal's orientation was reasonable.

Category 3. Statements of this category refer to the alignment of the animal or cuboid building in relation to the environment. Most of these statements were accompanied by gestures, which were often essential for the understanding. For the animals, examples of this category are "because he is standing this way (the child makes a movement from the back of the animal to its front)", "because he is looking there (the child shows the animal's viewing direction)". For the cuboid buildings statements like "this one (points to one cuboid) is there and this one (points to the other cuboid) is there" belong to this category.

For answers concerning tasks with side-views of the arrangements subcategories were identified. Statements that don't show an indication that the child noticed the difference between the side-views in the left-right-relation ("he looks that way (the child makes a movement from the back of the animal to his front)" or "this one (one cuboid) is there and the other one (the second cuboid) is next to it") were differentiated from statements that refer to the left-right-relation explicitly (for example "the drum is on the left side") or implicitly ("in this picture the head is on *that* side (points at the picture to the head) and not *over there* (points at the tail of the animal)" or "otherwise the blue one has to be *there* (puts the blue block to the other side of the second block)").

Category 4. Statements of this category refer to the viewing direction of the toy figurine. These statements do not give information about the specific view of the other observer and are therefore not further analyzed. Examples are "he is standing there" or "he looks this way (the child makes a movement from the figurine to the object)". For the cuboid buildings, this category also includes statements with relation-terms (“he is standing at the front”) because they cannot refer to the intrinsic alignment as for animals.

Category 9. Category 9 includes all statements that don’t refer to the task situation (the object, the pictures, or the toy figurine). They only confirm the solution, for example with statements like “I know it”, “Just because!”, or “because it looks that way”. Some children backed up such statements by rising up the picture or rotating it on the table until it matches the direction of the object.

Apart from these main categories, codes were assigned to tasks without justification, statements that could not be assigned to a category, or situations where the child answered “I don’t know”.

3.2.2 Results

The following section presents results concerning the solution rates of different task groups, the frequencies of errors types that occurred, and frequencies of explanation categories for these tasks.

Altogether, the children solved on average 22 of 32 items (M=22.31, SD=5.41). No significant difference between girls and boys could be found ($t(93)$ = -1.13, p>.05) and the four classes did not differ significantly ($F(3,91)$=0.26, p>.05). No significant correlation between the age of the children and the number of solved items could be found (r=-.006, p>.05).

Analysis of solution rates. In a first step, we compared solution rates between the different task-groups. Table 3 presents the solution rates separately for tasks that could be assigned to level 2 or level 3 while excluding tasks in which the other observer had the same view as the child.[1]

	solution rates[1]	errors[3]
level 2 total	**69 % (18 items)**	
front- and back-views	82 % (12 items)	52 % egocentric 48 % other
side-views of asymmetric objects	45 % (6 items)	25 % inversion 67 % egocentric 8 % other
level 3 total	**45 % (6 items)**	
side-views of symmetric objects	45 % (6 items)	42 % inversion 42 % egocentric 16 % other

Tab. 3: Solution rates and frequencies of mistakes for different item groups

Results showed that the average solution rate for tasks with front- and back-views was 82 %, whereas tasks with side-views of symmetric and asymmetric objects were solved on average by 45 % of the children.

We found significant differences in the solution rates between tasks with front- and back-views and tasks with side views of asymmetric objects ($t(93)$ = 14.33, p<.001) and between tasks with front- and back-views and tasks with side views of symmetric objects ($t(93)$ = 35.583, p<.001).[2] Solution rates between tasks with side views of symmetric and asymmetric objects did not differ significantly ($t(94)$ = -0.198, p>.05).

These findings indicated that the data can be modeled using a three-level model differentiating between level 2 and level 3. Concerning the level of solution rates only, however, it was not clear whether tasks involving side-views of asymmetric objects should be modeled as a differentiation of level 2 (initially proposed model) or whether they should be included in level 3 (model involving a differentiation according to the symmetry of the objects).

Analysis of errors. In a second step, we therefore considered the types of errors that the children made when giving wrong answers to tasks with side-views of asymmetric and symmetric objects.[3] Regarding asymmetric objects, the egocentric error was made most frequently dealing with items involving these objects (67 % of all errors). A confusion of side-views occurred in 25 % of all errors. Regarding symmetric objects, the egocentric error was made as frequently as the error to confuse side-views (42 % each of all the errors).

Since children were solving multiple items, the resulting data structure was nested with regard to the variable “child”. To account for this structure, a multinomial logistic multilevel model was computed in SPSS with “child did not make an error in the task” as reference category and type of the task (side-views of asymmetric objects vs. side-views of symmetric objects) as predictor (e.g., Heck, Thomas, & Tabata, 2012). Results showed that the odds of making an egocentric error versus not making an error were significantly reduced by 48.2 % (odds-ratio (OR)=0.525, p<.000) when solving tasks with side-views of symmetric objects compared to solving tasks with side-views of asymmetric objects. Concerning children making inversion errors versus not making an error, results showed that the type of the task also had a significant influence (OR=1.604, *p*=.032). Concerning this type of errors, solving tasks with side-views of symmetric objects increased the odds of making an inversion error versus no error by 60.4 %. Finally, concerning all other errors, the type of the task had no significant effect on the expected odds of making these errors versus no errors (OR=1.65, *p*=.10).

Altogether, results indicated that there were significant differences concerning making errors in tasks involving side-views of asymmetric objects and tasks involving side-views of symmetric objects, in particular for making egocentric errors or inversion errors.

These results indicated that when considering errors as indicators of cognitive processes rather than solutions rates alone, tasks requiring the children to find side-views of asymmetric objects are most likely not the same level (but rather level 2) as tasks requiring the children to find side-views of symmetric objects (level 3).

Analysis of explanations. Finally, to provide further evidence for this distinction, we considered the explanations for the correct solutions (Table 4). Results showed that most explanations for correct solutions in tasks involving front- and back-views could be assigned to category 1 or 2. For the animals, most of these explanations referred to the part of the animal that was seen by the other observer ("he sees the head", category 1). The remaining explanations referred to the specific side of the animal ("he looks to the front", category 2). In the case of the cuboid buildings, nearly all explanations described the front-back-relation of the two cuboids as seen by the other observer (category 1). Interestingly, most children did not mix up the relations with those from their own view when the other observer was opposite to the child. They described the relation between the cuboids from the other point of view, although this description did not correspond with their own view.

	category 1	**category 2**	**category 3**	**other**
level 2 total				
front- and back-views	57 %	21 %	4 %	18 %
side-views of asymmetric objects	46 %	10 %	14 %	30 %
level 3 total				
side-views of symmetric objects	6 %	12 %	34 %	48 %

Tab. 4: Frequencies of explanation-categories for different item groups[4]

Concerning tasks with side-views of asymmetric objects, most explanations (46 % of the correct solutions) referred to the specific part of the object that caused the asymmetry (for example a risen leg or a protruding part of the cuboid building) in combination with the front-back-relation (for example "it must be this photo because there the risen leg is at the front") (category 1). Not all children used the words "front" or "back" explicitly, but even without these words the reference to this relation was obvious ("the risen leg is at his side", "the cuboids are equally near").

Concerning side-views of symmetric objects, only 6 % of the explanations could be assigned to category 1 and most of them did not distinguish sufficiently between the two side-views ("you can see one of the eyes and the tail fin of the dolphin"). In contrast, most explanations (34 % of the correct solutions) could be assigned to category 3. They referred implicitly or explicitly to the object's orientation. However, nearly half of these explanations were very general and did not refer to the difference of the two side-views ("he is looking in my direction", "this one is here, and that one is over there").

Taking the explanations of all categories together it can be resumed that only 25 % clearly distinguished between the two side-views of symmetrical objects whereas more than twice as many clear discriminations could be observed at the tasks with side views of asymmetric objects (53 %).

Results of a multinomial logistic multilevel model with "child did not give an explanation (because the task was not solved correctly)" as reference category and type of the task (side-views of asymmetric objects vs. side-views of symmetric objects) as predictor showed that the predicted odds that a child solving tasks including symmetric objects is giving a category 1-explanation vs. not giving an explanation are 0.120 of the odds for the child solving tasks including asymmetric objects. In other words, the odds of giving a category 1-explanation vs. not giving an explanation were significantly reduced by 88 % (OR=0.120, *p*<.001) for children solving the tasks involving symmetric objects compared to tasks involving asymmetric objects.

Turning to giving a category 2-explanation vs. no explanation, the results showed that the type of the task solved had no significant effect on the explanation category given (OR=1.337, *p*=0.307).

Concerning giving a category 3-explanation vs. not giving an explanation, the results indicated that the type of the task had a significant effect on the explanation category given (OR=2.588, p<.001). In this case, the odds of giving a category 3-explanation vs. not giving an explanation are multiplied by 2.588 (or significantly increased by 159 %) for children solving tasks involving symmetric objects versus children solving tasks involving asymmetric objects.

Altogether, results on the level of explanations also indicated that there were significant differences concerning explaining correct solutions of tasks involving side-views of asymmetric objects and tasks involving side-views of symmetric objects.

4. Discussion and Conclusions

Spatial perspective-taking refers to a conglomerate of cognitive abilities that enable individuals to perform a mental change into another observer's position. Tasks requiring spatial perspective-taking can be designed in different ways. Different tasks and task characteristics, however, are unequally challenging the children who are asked to solve such tasks. A model describing perspective-taking in greater detail could help teachers to classify different tasks concerning to their demands. The model of Flavell et al. (1981) is a suitable model to describe very basic level-1-perspective-taking abilities at age three to six. For older children, this model is too undifferentiated to reflect the development of perspective-taking. It cannot explain why some level-2-tasks can be solved easily by children at age six to ten, while others remain difficult.

Rosser et al. (1985) suggested a further elaboration of level 2 into two different levels. In this study, we addressed this proposal by showing that this distinction allows for interpreting research results in greater detail. Moreover, a differentiation of level 2 was found to be suitable for categorizing tasks and describing students' abilities in a differentiated way.

Results of our theoretical analyses showed that a three-level model is useful to distinguish different demands of tasks depending on the relations that have to be coordinated and considered to solve the task successfully. Level-2-tasks can be solved by considering the external relation between observer and object, for example by analyzing which key feature of the object is nearest to the other observer in a front-back-task. Level-3-tasks afford to coordinate the intrinsic object relations and cannot be reinterpreted in terms of key features of the object. Tasks with symmetries could be, for example, classified as level-3-tasks, because two or more views are symmetric to each other and can only be discriminated by considering the intrinsic left-right-relation.

Results of our empirical analyses indicated that a three-level model is suitable to describe first graders' solutions in a set of perspective-taking tasks. Overall, the results showed that taking only solutions rates into account, did not explain the data sufficiently well. Only by taking additionally a cognitive perspective we were able to do so. That is, although the solution rates did not differ between tasks with side-views of symmetric objects (level 3) and similar tasks with asymmetric objects (level 2), the explanations of the children showed that it is more difficult for them to refer to the left-right-relation of symmetric objects than to the front-back-relation or to key features of asymmetric objects.

The results showed that tasks with front- and back-views of animals or views, in which one cuboid is seen in front of the other one, are solved best by children at the age of 5 to 7. Almost all children could also explain their solution by referring to what part of the animal is seen in the front or by describing the front-back-relation of the cuboids. The children had no problems to describe this external relation between the observer and the object from the other observer's view in these tasks, even if this description did not correspond with their own view. So, they were not arguing in an egocentric manner.

Tasks with side-views of asymmetric objects (level 2) and tasks with side-views of symmetric objects (level 3) did not differ in their solution rates. With symmetric objects, however, the children more often interchanged the two side-views that could only be discriminated by considering the left-right-orientation. The explanations of the children showed that they tended to refer to key features of the object or to the front-back-relation if possible. Only few children managed to refer to the left-right-relation clearly if that was necessary. But many children showed that they were aware of the differences between side-views of symmetric objects in the left-right-relation.

Since the solution rates, the errors, and the explanations differed between the two task groups of level 2 (front- and back-views and side-views of asymmetric objects), we suggest to differentiate level 2 into two sublevels: On the one hand, level-2a-tasks in which the other observer's view differs from other alternatives in what part of the object is seen. These tasks can be justified by naming the specific key feature of the object that is seen by the other observer. On the other hand, level-2b-tasks in which different views can only be discriminated by referring to the front-back-relation of the object, such as in tasks involving two pictures showing similar sides of that object. This additional differentiation of the perspective-taking model has to be discussed further and needs to be validated in another study.

There are also several limitations of this study. In particular, the analyses of our data focused on the explanations of the children. This could have disadvantaged children who had not enough verbal abilities to describe what they thought. Moreover, the requirement to explain a solution did not fit the holistic strategy that is described as one powerful strategy for spatial tasks. Research methods that do not need verbalizations such as eye tracking (e.g., Ferguson, Apperly, & Cane, 2017) might overcome this limitation in future studies.

Altogether, the findings show that the further differentiation of Flavell et al.'s (1981) level 2, as suggested by Rosser et al. (1985), can be validated both from a theoretical and an empirical point of view. Our theoretical analyses and the results of the interview study with 95 children indicated that perspective-taking abilities could be modeled using a three-level model with the proposed sublevels. The characteristics of level 2 and level 3, however, should be addressed and refined in future studies.

Notes

[1] Items, where the other observer has the same view as the child, were excluded for this calculation. They were solved correctly by nearly all children (93 %).

[2] Data missing for one child in the front-and-back-view task.

[3] For the analysis of errors, we excluded the items where the figure has the same view as the child or was opposite to it because in this situation no egocentric error could occur or was not unambiguously discriminable from the inversion of side-views.

[4] Explanations of category 4 and 9 are not analyzed explicitly in this table because they do not give information about the specific view of the other observer. Consequently, no relational sight becomes visible.

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Literature

Cheng, Y.-L., & Mix, K. S. (2014). Spatial Training Improves Children's Mathematics Ability. *Journal of Cognition and Development, 15*(1), 2–11. doi: 10.1080/15248372.2012.725186.

Coie, J. D., Costanzo, P., & Farnill, D. (1973). Specific transitions in the development of spatial perspective-taking ability. *Developmental Psychology, 9*(2), 167–177.

Cox, M. V. (1977a). Perspective ability: The other observer in the task. *Perceptual and Motor Skills, 44*(1), 76.

Cox, M. V. (1977b). Perspective ability: The relative difficulty of the other observer's viewpoints. *Journal of Experimental Child Psychology, 24*(2), 254–259.

Eiser, C. (1974). Recognition and inference in the coordination of perspectives. *British Journal of Educational Psychology, 44*(3), 309–312.

Eiser, C. (1976). Questions children ask about spatial arrays: An analysis of the processes involved in coordinating perspectives. *British Journal of Educational Psychology, 46*(2), 203–211.

Ferguson, H. J., Apperly, I., & Cane, J. E. (2017). Eye tracking reveals the cost of switching between self and other perspectives in a visual perspective-taking task. *Quarterly Journal of Experimental Psychology*, *70*(8), 1646-1660.

Flavell, J. H., Everett, B. A., Croft, K., & Flavell, E. R. (1981). Young children's knowledge about visual perception: Further evidence for the Level 1–Level 2 distinction. *Developmental Psychology, 17*(1), 99–103.

Frick, A., Möhring, W., & Newcombe, N. S. (2014). Picturing perspectives: Development of perspective-taking abilities in 4- to 8-year-olds. *Frontiers in Psychology, 5*, 1–7. doi: 10.3389/fpsyg.2014.00386.

Graß, K., Krammer, G. (2018). Direkte und indirekte Einflüsse der Raumvorstellung auf die Rechenleistungen am Ende der Grundschulzeit. *Journal für Mathematik-Didaktik, 39*(1) 43–67.

Gzesh, S. M., & Surber, C. F. (1985). Visual perspective-taking skills in children. *Child Development, 56*(5), 1204–1213.

Heck, R. H., Thomas, S., & Tabata, L. (2012). *Multilevel modeling of categorical outcomes using IBM SPSS*. New York, NY: Routledge Academic.

Hegarty, M., & Waller, D. (2005). Individual differences in spatial abilities. In P. Shah & A. Miyake (Eds.), *The cambridge handbook of visuospatial thinking* (pp. 121–169). New York, NY: Cambridge University Press.

Hobson, R. P. (1980). The question of egocentrism: The young child's competence in the coordination of perspectives. *Journal of Child Psychology and Psychiatry and Allied Disciplines, 21*(4), 325–331.

Hughes, M., & Donaldson, M. C. (1979). The use of hiding games for studying the coordination of viewpoints. *Educational Review, 31*(2), 133–140.

Huttenlocher, J., & Presson, C. C. (1979). The coding and transformation of spatial information. *Cognitive Psychology, 11*(3), 375–394.

Ives, S. W., & Rakow, J. (1983). Children's use of feature descriptions to solve spatial perspective and rotation problems. *British Journal of Educational Psychology, 53*(2), 143–151.

Jacobsen, T. L., & Waters, H. S. (1985). Spatial Perspective Taking: Coordination of Left-Right and Near-Far Spatial Dimensions. *Journal of Experimental Child Psychology, 39*(1), 72–84.

Laurendeau, M., & Pinard, A. (1970). *The development of the concept of space in the child.* New York: International University Press.

Linn, M. C., & Petersen, A. C. (1985). Emergence and characterization of sex differences in spatial ability: A meta-analysis. *Child development*, *56*(6), 1479–1498.

Masangkay, Z. S., McCluskey, K. A., McIntyre, C. W., Sims-Knight, J., Vaughn, B. E., & Flavell, J. H. (1974). The early development of inferences about the visual percepts of others. *Child Development, 45*(2), 357–366.

Mayring, P. (2010). *Qualitative Inhaltsanalyse: Grundlagen und Techniken* (11. Auflage). Weinheim: Beltz.

Mulligan, J. (2015). Looking within and beyond the geometry curriculum: Connecting spatial reasoning to mathematics learning. *ZDM: Mathematics Education, 47*(3), 511–517. doi: 10.1007/s11858-015-0696-1.

Newcombe, N. S. (1989). The development of spatial perspective taking. In H. W. Reese (Ed.), *Advances in Child Development and Behavior* (Vol. 22, pp. 203–247). New York: Academic Press.

Niedermeyer, I. (2015). *Räumliche Perspektivübernahme am Schulanfang: Eine Interviewstudie zum Einfluss der Symmetrie*. Münster u.a.: Waxmann.

Niedermeyer, I., & Ruwisch, S. (2014). The influence of symmetric objects on spatial perspective-taking: An interview-study with young elementary school children. In P. Liljedahl, S. Oesterle, C. Nicol, & D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36* (Vol. 4, pp. 305–312). Vancouver, Canada: PME.

Nigl, A. J., & Fishbein, H. D. (1974). Perception and conception in coordination of perspectives. *Developmental Psychology, 10*(6), 858–866.

Piaget, J., & Inhelder, B. (1971). *The child's conception of space* (4. impr.). London: Routledge & Kegan Paul.

Presson, C. C. (1982). Strategies in spatial reasoning. *Journal of Experimental Psychology: Learning, Memory and Cognition, 8*(3), 243–251.

Rosser, R. A., Ensing, S. S., Mazzeo, J., & Horan, P. F. (1985). Visual Perspective Taking in Children: Further Ramifications of an Information-Processing Model. *Journal of Genetic Psychology, 146*(3), 379–387.

Sinclair, N., & Bruce, C. D. (2015). New opportunities in geometry education at the primary school. *ZDM: Mathematics Education, 47*(3), 319–329. doi: 10.1007/s11858-015-0693-4.

van den Heuvel-Panhuizen, M., Elia, I., & Robitzsch, A. (2015). Kindergartners' performance in two types of imaginary perspective-taking. *ZDM: Mathematics Education, 47*(3), 345–362. doi: 10.1007/s11858-015-0677-4.

Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., Newcombe, N. S., Filipowicz, A. T., & Chang, A. (2014). Deconstructing building blocks: Preschoolers' spatial assembly performance relates to early mathematical skills. *Child Development, 85*(3), 1062–1076. doi: 10.1111/cdev.12165.

Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial Ability for STEM Domains: Aligning over 50 Years of Cumulative Psychological Knowledge Solidifies Its Importance. *Journal of Educational Psychology, 101*(4), 817–835. doi: 10.1037/a0016127.

Walker, L. D., & Gollin, E. S. (1977). Perspective role-taking in young children. *Journal of Experimental Child Psychology, 24*(2), 343–357.

Wraga, M., Creem, S. H., & Proffitt, D. R. (2000). Updating displays after imagined object and viewer rotations. *Journal of Experimental Psychology: Learning, Memory and Cognition, 26*(1), 151–168. doi: 10.1037//0278-7393.26.1.151.

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