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Research article

Fuzzification of the Distributed Activation Energy Model Using the Fuzzy Weibull Distribution

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Abstract

This study focuses on the influence of some of the relevant parameters of biomass pyrolysis on a fuzzified solution of the Distributed Activation Energy Model (DAEM) due to randomness and inaccuracy of data. The study investigates the fuzzified Distributed Activation Energy Model using the fuzzy Weibull distribution. The activation energy, frequency factor, and distribution variables of the 3-parameter Weibull analysis are converted into a non-crisp set. The expression for the fuzzy sets, and their α-cut are discussed with an initial distribution for the activation energies following the Weibull distribution function. The thermo-analytical data for pine needles is used to illustrate the methodology to exhibit the fuzziness of some of the parameters relevant to biomass pyrolysis.

1. Introduction

Many engineering systems that need to be controlled or simply are analyzed by the introduction of an indefinite value for parameters in a prototype. For a concrete mathematical problem, it is necessary to tackle any anomaly in the given data. The basis of incompatibility reveals that our faculty for envisaging some precise and relevant axioms becomes misleading while the critical level of the compounded complexity of the system is not reached, and, proceeding further, the precision and significance do not co-exist **[1]**. The ramifications of fallacy in a deterministic model is of the least importance. The conundrum of this state is whether substituting the rigid imprecise data for fixed values will influence the outcome. There is much evidence **[1]**-**[3]** that tells us not to rely upon the hammer principle (i.e. when you only have a hammer, you want everything at hand to be a nail). Sometimes incoherent outcomes are obtained from models when the arbitrarily imprecise variables are assumed to be the parameters of the model. Uncertainty in the measurement of variables is a problem if it shrinks your ability to dichotomize crucial choices. However, the stochastic approach makes a model decision difficult **[4]**, although qualitatively, there are various facets of vagueness which are beyond the probabilistic scope. Under certain circumstances where accuracy of concepts is subjected to debrief, veracity of statements and judgements become alienated through prediction of events. Consequently, this led to the discovery of a new methodology of incorporating this non-stochastic imprecision into mathematical models. Eventually, the concept of fuzzy set theory [4], **[5]**, a theory of belief and evidence **[6]**-**[8]**, was introduced to evaluate of randomness in any given data.

The aim of the present study is to use this theory to obtain a non-stochastic outcome for the Distributed Activation Energy Model (DAEM). The kinetic parameters as well as distribution parameters of the Weibull distribution were fuzzified using a fuzzy relationship.

The present paper is presented as follows: Section 2 introduces the evolution of the DAEM, the Weibull distribution, the numerical solution of the DAEM using asymptotic expansion and fuzzy sets; Section 3 involves application of thermo-analytical data from pine needle pyrolysis; Section 4 illustrates the numerical implementation of the fuzzy system on the predefined problem of the kinetics of biomass; and the last section is the conclusion of the study.

2. Theory

2.1 Model-based approach

For this investigation of fuzzy set theory, the Distributed Activation Energy Model (DAEM) or Multiple Reaction Model (MRM) is adopted to evaluate the randomness in estimating kinetic parameters. Equation (1) represents the solid-state kinetic reaction:

$$\frac{dv_i}{dt} = A_{0i} \exp\left(-\frac{E_i}{RT}\right)(v^*{}_i - v_i) \tag{1}$$

Here subscript i represents one of several constituents of biomass, v_i (mg) is the total release mass of the ith constituent, t (*s*) is time, A_{0i} (min^{-1}) is the frequency factor, E_i (kJ mol^{-1}) is the activation energy, *R* (kJ/mol-K) is the gas constant and *T* is the absolute temperature.

If the number of decomposition reaction steps is high, it can be presumed that the activation energies of these reactions follow a continuous distribution function, and the reactions can be expressed as a function of the activation energy.

$$dv^* = v^* f(E)\, dE \tag{2}$$

Equation (2) shows the change in the volatile fraction of biomass with respect to the distribution function.

Mainly, *f (E)* is assumed to be a Gaussian distribution, while the validity of the distribution of activation energy for asymmetric can be examined with the help of a positively skewed function, therefore, the '3-parameter type' Weibull distribution function is assumed i.e. a Gaussian function **[9]**.

The final expression for the DAEM for non-isothermal time dependent temperature regime is obtained using equations (1) and (2), giving the following:

$$v = \int_0^\infty \left[exp\left(\int_{T_0}^{T} \frac{-A}{\theta} exp\left(\frac{-E}{RT}\right) dT\right)\right] f(E) dE \tag{3}$$

where, θ (K/min) corresponds to the heating rate, *E* (kJ/mol) represents the activation energy and *A* is the frequency factor (s^{-1}). A constant value of the frequency factor (*A*) is assumed for every decomposition reaction with various activation energies **[10]**. The value of the pre-exponential factor can also be expressed as a function of activation energy or temperature (the fluctuant pre-exponential factor leads to different algorithms to treat the DAEM. Corresponding references ought to be mentioned).

The initial Weibull distribution of the activation energy can be expressed as:

$$f(E) = \frac{\lambda}{\eta}\left(\frac{E-\gamma}{\eta}\right)^{\lambda-1} exp\left[\left(\frac{E-\gamma}{\eta}\right)^{\lambda}\right] \tag{4}$$

where η is the scale parameter; λ is the shape parameters; γ is the threshold or location parameter and $0 \le \gamma \le E$, $0 < \lambda$, $0 < \eta$. λ is dimensionless and η, γ, and *E* are expressed in kJ/mol.

The mean of the distribution is equal to the mean activation energy and is given by:

$$\mu = E_0 = \gamma + \eta\Gamma\left(\frac{1}{\lambda}+1\right) \tag{5}$$

The variance of the distribution is given by:

$$\sigma^2 = \eta^2\Gamma\left(\frac{2}{\lambda}+1\right) - \eta^2\Gamma^2\left(\frac{1}{\lambda}+1\right) \tag{6}$$

where, $\Gamma(2/\lambda+1)$ is the Gamma function.

The Weibull distribution has some interesting properties and generates a variety of distributions **[11]**. For $\lambda = 1$, the Weibull distribution coincides with the exponential distribution. The Weibull distribution curves are positively skewed for values of $\lambda > 1$. As the value of λ increases, the Weibull distribution tends to approach the Gaussian distribution more and more closely **[12]**. Selection of the threshold value for the activation energy γ, implies that reactions with activation energy less than that of γ will not occur. Thus, the lower limit of the outer *dE* integral in equation (3) should be replaced with $E = \gamma$. Then, the non-isothermal DAEM involving the Weibull distribution is obtained and given as follows:

$$v = \int_{\gamma}^{\infty} \left[exp\left(\int_{T_0}^{T} \frac{-A}{\theta} exp\left(\frac{-E}{RT}\right) dT \right) \right] \frac{\lambda}{\eta} \left(\frac{E-\gamma}{\eta}\right)^{\lambda-1} exp\left[\left(\frac{E-\gamma}{\eta}\right)^{\lambda} \right] dE \quad (7)$$

2.2 Asymptotic solution of the Distributed Activation Energy Model

Approximation to the double exponential term in equation (7) is considered first.

$$DExp = exp\left[-\int_{0}^{t} \frac{A}{\theta} exp\left(-\frac{E}{R\theta l}\right) dl \right] \quad (8)$$

To proceed in a systematic manner, it is useful to consider the typical values of the parameters and functions on which it depends. The range of the frequency factor (A) is 10^{10}-10^{13} s^{-1}, whereas the activation energies lie in the domain of 100-300 kJ mol^{-1}. The temperature scale defined for pyrolysis is in the range of 293 to 900 K. In order to demonstrate the approach, the non-isothermal regime was used for our problem.
where,

$$T = \theta t \quad (9)$$

θ = Heating rate (°C min^{-1})
t = time (min)

The double integral in equation (8) can be approximated with the help of an asymptotic scheme for solving the double exponential part of the DAEM, where the parameter '*E/Rθl*' is assumed to be large and thus, the integral is approximated using Taylor series expansion around the maximum value of the function *θl* .

$$exp\left[-\int_{0}^{t} \frac{A}{\theta} exp\left(-\frac{E}{R\theta l}\right) dl \right] = \exp\left[\exp\left(\frac{-AR\theta t^2}{E} e^{-\frac{E}{R\theta t}} \right) \right] \text{ as } \frac{E}{R\theta t} \rightarrow \infty \quad (10)$$

assuming the typical values of $E/RT \sim 10$, where $A.t = 10^{10}$. The large values of both parameters make the function vary rapidly with *E*. Rewriting equation (10) as:

$$DExp \sim exp\left[-exp\left(\frac{E_s - E}{E_w} \right) \right]$$

The function, $(-AR\theta t^2/E)\, e^{-E/R\theta t}$, inside the exponential is converted into a step size variable E_w and the central value of the activation energy E_s. For *E* much lower than the stationary value, E_s , the function is nearly zero; whereas for *E* much higher than E_s, the function approaches one. The function changes from zero to one in a range of *E* values with a step size of approximately E_w of E_s . Let,

$$g(E) = \left(\frac{E_s - E}{E_w} \right)$$

where,

$$g(E) = \left(\frac{E_s - E}{E_w} \right) \quad (11)$$

Since the behavior in the neighborhood of E_s is the sole interest, equation (11) is expanded with the help of a Taylor series:

$$g(E) \sim g(E_s) + (E - E_s)\, g'(E_s) + \ldots \quad (12)$$

The initial value problem of the function *g (E)* is stated as:

$g(E_s) = 0$ and $g'(E_s) = (-1/E_w)$

Putting these boundary conditions into equation (12), we get

$$E_s = R\theta t\, W(\tau) \quad (13)$$

$$E_w = \left(\frac{R\theta t E_s}{R\theta t + E_s} \right) \quad (14)$$

where $\tau = A.t$; (Time is rescaled as a product of frequency factor and time).

Here $W(\tau)$ represents the Lambert *W* function and is considered to one of the roots of equation (11)

$We^{W} = x$

Approximation of the Lambert *W* function depends on the value of *x* **[13]** and is given by

$$x - x^2 \sim W(x), \qquad x \ll 1$$

$$ln\left(\frac{x}{ln\left(\frac{x}{\ln(x)}\right)} \right) \sim W(x), \qquad 1 \ll x$$

In order to apply the asymptotic approach, the Weibull distribution is assumed as the initial distribution *f (E)*, centered around E_0 with the standard deviation σ.

$$X = \int_{\Upsilon}^{\infty} \frac{\beta}{\eta} \left(\frac{E - \Upsilon}{\eta} \right)^{(\beta-1)} exp(h(E))\, dE \quad (15)$$

where,

$$h(E) = \left\{ -exp\left(\frac{E_s - E}{E_w} \right) - \left(\frac{E - \Upsilon}{\eta} \right)^{\lambda} \right\}$$

Energy is rescaled into a non-dimensional factor by *y*, which can be expressed as:

$$y_s = \frac{E_s}{\Upsilon}, y = \frac{E}{\Upsilon}, and\ y_w = \frac{E_w}{\Upsilon}$$

where, $\alpha = \Upsilon/\eta$

$$h(y) = \left\{ -exp\left(\frac{y_s - y}{y_w} \right) - \left(\alpha(y-1) \right)^{\lambda} \right\} \quad (16)$$

$$X = \int_1^{\infty} \lambda\alpha^{\lambda}(y-1)^{(\lambda-1)} exp(h(y))\, dy \tag{17}$$

Putting *k = y - 1* into equation (17),we have:

$$X = \lambda\alpha^{\lambda} \int_0^{\infty} k^{(\lambda-1)} exp(h(k+1))\, dk$$

Wide distribution case

A distribution is said to be a wide distribution when the relative width of the distribution function is wider than that of DExp. It implies that $k_w \sqrt{\alpha} \ll 1$ **[14]**-**[17]**.

$$U(k) = \begin{cases} 0, & k < k_s \\ 1, & k \geq k_s \end{cases}$$

Using a step-function in equation (17), we have:

$$X = \beta\alpha^{\beta} \int_0^{\infty} k^{(\lambda-1)} \left[exp\left\{-exp\left(\frac{k_s-k}{k_w+1}\right)\right\} - U(k-k_s)\right] \exp\{-(\alpha k)^{\lambda}\}\, dk + \int_{k_s}^{\infty} \beta\alpha^{\lambda} k^{(\lambda-1)} \exp\{-(\alpha k)^{\lambda}\} dk$$

$$X = \left(1 - G(k)\right) + \beta\alpha^{\lambda} \int_0^{\infty} k^{(\lambda-1)} \left[exp\left\{-exp\left(\frac{k_s-k}{k_w+1}\right)\right\} - U(k-k_s)\right] \exp\{-(\alpha k)^{\lambda}\}\, dk$$

It is clear from the expression above that the (1-G(k)) term in the integral represents a complementary distribution function, hence it can be estimated easily. Whereas the second integral is multiplied by a function that is very small everywhere except in the vicinity of k_s. Therefore, this can be approximated using a Taylor series expansion around *k = ks* .

Let $\left(\frac{k-k_s}{k_w+1}\right) = x$ and $f(x) = k^{(\lambda-1)} \exp\{-(\alpha k)^{\lambda}\}$

Using a Taylor series, *f (x)* is expanded around $k = k_s$, giving the following:

$$f(k) = (k_s)^{(\lambda-1)}\exp(-(\alpha k_s)^{\beta})\Big[1 - \frac{x(k_w+1)}{k_s}\left(\lambda - \lambda(\alpha k_s)^{\lambda} - 1\right) + \left(\frac{x(k_w+1)}{k_s}\right)^2 \left(\lambda^2\left(-3(\alpha k_s)^{\lambda} + (\alpha k_s)^{2\lambda} + 1\right) + 3\lambda\left((\alpha k_s)^{\lambda} - 1\right) + 2\right) - \left(\frac{x(k_w+1)}{k_s}\right)^3 \left(\alpha^2\lambda^4(-k_s{}^2)\left((\alpha k_s)^{\lambda} - 1\right) + \lambda^3\left(-(\alpha k_s)^2 - 7(\alpha k_s)^{\lambda} + 3(\alpha k_s)^{2\lambda} + 1\right) - 3\lambda^2\left(-6(\alpha k_s)^{\lambda} + (\alpha k_s)^{2\lambda} + 2\right) - 11\lambda\left((\alpha k_s)^{\lambda} - 1\right) - 6\right)\Big]$$

$$X = \left(1 - G(k)\right) + \lambda\alpha^{\lambda} \int_0^{\infty} \left[exp\left\{-exp\left(\frac{k_s-k}{k_w+1}\right)\right\} - U(k-k_s)\right] f(k)dk$$

where $G(k) = (1 - \exp(\alpha k)^{\lambda}$

Simplifying equation (18), we have:

$$X = \left(1 - G(k)\right) + (k_s)^{(\lambda-1)}\exp(-(\alpha k_s)^{\lambda})\Big[A_0 - \frac{(k_w+1)}{k_s}\left(\lambda - \lambda(\alpha k_s)^{\lambda} - 1\right)A_1 + \left(\frac{(k_w+1)}{k_s}\right)^2 A_2\left(\lambda^2\left(-3(\alpha k_s)^{\lambda} + (\alpha k_s)^{2\lambda} + 1\right) + 3\lambda\left((\alpha k_s)^{\lambda} - 1\right) + 2\right) - \left(\frac{(k_w+1)}{k_s}\right)^3 A_3\left(\alpha^2\lambda^4(-k_s{}^2)\left((\alpha k_s)^{\lambda} - 1\right) + \lambda^3\left(-(\alpha k_s)^2 - 7(\alpha k_s)^{\lambda} + 3(\alpha k_s)^{2\lambda} + 1\right) - 3\lambda^2\left(-6(\alpha k_s)^{\lambda} + (\alpha k_s)^{2\lambda} + 2\right) - 11\lambda\left((\alpha k_s)^{\lambda} - 1\right) - 6\right)\Big]$$

$A_0 \approx -0.5772$, $A_1 \approx -0.98906$, $A_2 \approx -1.81496$, $A_3 \approx -5.89037$.

where the remaining integrals are evaluated by

$$A_i = \int_{-\infty}^{\infty} x^i \left(exp(-x) - U(x)\right) dx\, , i = 0,1,2,3 \ldots$$

2.3 Fuzzy set

Fuzzy set theory was proposed in 1965 **[1]**, relying on the fact that the degree of membership and non-membership should not be more than one. In practical situations an object may or may not be a member of set A to a certain degree and it is not to be ousted completely. In other sense, there is degree of belongingness of an object in the family.

The motive of introducing fuzzy sets is to demarcate gradual membership to a set without sharp boundaries. The subtlety of a fuzzy set is that it somehow resembles the human representation of reality rather than a clear-cut representation. Philosophical argument concerning the existence of an entity with the help of the faculty of dichotomizing is non-realistic, and practicality is not part of fuzzification. In a fuzzy set, the extent of membership counts in real numbers varies from 0 to 2 rather than end points. In other words, a fuzzified variable of a set A is not the null set, but a membership function ξ :A → [0,1].

2.4 Fuzzy Weibull Distribution

Due to vagueness and inaccuracy in data sets, the estimation of precise values for distribution parameters (*λ, η*) and kinetic parameters can often become very difficult. To handle this situation, shape and scale as well as kinetic parameters can be replaced by the trapezoidal fuzzy numbers $\tilde{\lambda}$ $\tilde{\eta}$, $\tilde{A}$ and $\tilde{E}$ In this case, the fuzziness of a distribution is given by:

$$f(\tilde{E}) = \frac{\tilde{\lambda}}{\tilde{\eta}}\left(\frac{\tilde{E}-\gamma}{\tilde{\eta}}\right)^{\tilde{\lambda}-1} exp\left[\left(\frac{\tilde{E}-\gamma}{\tilde{\eta}}\right)^{\tilde{\lambda}}\right]$$

or

$$v = \int_{\gamma}^{\infty}\left[exp\left(\int_{T_0}^{T}\frac{-\tilde{A}}{\theta}exp\left(\frac{-\tilde{E}}{RT}\right)dT\right)\right]\frac{\tilde{\lambda}}{\tilde{\eta}}\left(\frac{\tilde{E}-\gamma}{\tilde{\eta}}\right)^{\tilde{\lambda}-1} exp\left[\left(\frac{\tilde{E}-\gamma}{\tilde{\eta}}\right)^{\tilde{\lambda}}\right]dE \qquad (19)$$

The general expression for fuzzy probability is given as:

$$\tilde{P}(n < v < m)[\alpha] = \left\{\int_{n}^{m}\left[exp\left(\int_{T_0}^{T}\frac{-A}{\theta}exp\left(\frac{-E}{RT}\right)dT\right)\right]\frac{\lambda}{\eta}\left(\frac{E-\gamma}{\eta}\right)^{\lambda-1} exp\left[\left(\frac{E-\gamma}{\eta}\right)^{\lambda}\right]dE \middle| E \in \tilde{E}\,[\alpha]\right\} = \left[P^L[\alpha], P^U[\alpha]\right],$$

$$\tilde{P}(n < v < m)[\alpha] = \left\{\int_{n}^{m}\left[exp\left(\int_{T_0}^{T}\frac{-A}{\theta}exp\left(\frac{-E}{RT}\right)dT\right)\right]\frac{\lambda}{\eta}\left(\frac{E-\gamma}{\eta}\right)^{\lambda-1} exp\left[\left(\frac{E-\gamma}{\eta}\right)^{\lambda}\right]dE \middle| A \in \tilde{A}\,[\alpha]\right\} = \left[P^L[\alpha], P^U[\alpha]\right]$$

$$\tilde{P}(n < v < m)[\alpha] = \left\{\int_{n}^{m}\left[exp\left(\int_{T_0}^{T}\frac{-A}{\theta}exp\left(\frac{-E}{RT}\right)dT\right)\right]\frac{\lambda}{\eta}\left(\frac{E-\gamma}{\eta}\right)^{\lambda-1} exp\left[\left(\frac{E-\gamma}{\eta}\right)^{\lambda}\right]dE \middle| \lambda \in \tilde{\lambda}\,[\alpha]\right\} = \left[P^L[\alpha], P^U[\alpha]\right],$$

$$\tilde{P}(n < v < m)[\alpha] = \left\{\int_{n}^{m}\left[exp\left(\int_{T_0}^{T}\frac{-A}{\theta}exp\left(\frac{-E}{RT}\right)dT\right)\right]\frac{\lambda}{\eta}\left(\frac{E-\gamma}{\eta}\right)^{\lambda-1} exp\left[\left(\frac{E-\gamma}{\eta}\right)^{\lambda}\right]dE \middle| \eta \in \tilde{\eta}\,[\alpha]\right\} = \left[P^L[\alpha], P^U[\alpha]\right]$$

for all α, where,

$$P^L = min\left\{\left[exp\left(\int_{T_0}^{T}\frac{-A}{\theta}exp\left(\frac{-E}{RT}\right)dT\right)\right]\frac{\lambda}{\eta}\left(\frac{E-\gamma}{\eta}\right)^{\lambda-1}exp\left[\left(\frac{E-\gamma}{\eta}\right)^{\lambda}\right]|E\in\tilde{E}[\alpha]\right\}, P^U = max\left\{\left[exp\left(\int_{T_0}^{T}\frac{-A}{\theta}exp\left(\frac{-E}{RT}\right)dT\right)\right]\frac{\lambda}{\eta}\left(\frac{E-\gamma}{\eta}\right)^{\lambda-1}exp\left[\left(\frac{E-\gamma}{\eta}\right)^{\lambda}\right]|E\in\tilde{E}[\alpha]\right\}$$

$$P^L = min\left\{\left[exp\left(\int_{T_0}^{T}\frac{-A}{\theta}exp\left(\frac{-E}{RT}\right)dT\right)\right]\frac{\lambda}{\eta}\left(\frac{E-\gamma}{\eta}\right)^{\lambda-1}exp\left[\left(\frac{E-\gamma}{\eta}\right)^{\lambda}\right]|A\in\tilde{A}[\alpha]\right\}, P^U = max\left\{\left[exp\left(\int_{T_0}^{T}\frac{-A}{\theta}exp\left(\frac{-E}{RT}\right)dT\right)\right]\frac{\lambda}{\eta}\left(\frac{E-\gamma}{\eta}\right)^{\lambda-1}exp\left[\left(\frac{E-\gamma}{\eta}\right)^{\lambda}\right]|A\in\tilde{A}[\alpha]\right\}$$

$$P^L = min\left\{\left[exp\left(\int_{T_0}^{T}\frac{-A}{\theta}exp\left(\frac{-E}{RT}\right)dT\right)\right]\frac{\lambda}{\eta}\left(\frac{E-\gamma}{\eta}\right)^{\lambda-1}exp\left[\left(\frac{E-\gamma}{\eta}\right)^{\lambda}\right]|\lambda\in\tilde{\lambda}[\alpha]\right\}, P^U = max\left\{\left[exp\left(\int_{T_0}^{T}\frac{-A}{\theta}exp\left(\frac{-E}{RT}\right)dT\right)\right]\frac{\lambda}{\eta}\left(\frac{E-\gamma}{\eta}\right)^{\lambda-1}exp\left[\left(\frac{E-\gamma}{\eta}\right)^{\lambda}\right]|\lambda\in\tilde{\lambda}[\alpha]\right\}$$

$$P^L = min\left\{\left[exp\left(\int_{T_0}^{T}\frac{-A}{\theta}exp\left(\frac{-E}{RT}\right)dT\right)\right]\frac{\lambda}{\eta}\left(\frac{E-\gamma}{\eta}\right)^{\lambda-1}exp\left[\left(\frac{E-\gamma}{\eta}\right)^{\lambda}\right]|\eta\in\tilde{\eta}[\alpha]\right\}, P^U = max\left\{\left[exp\left(\int_{T_0}^{T}\frac{-A}{\theta}exp\left(\frac{-E}{RT}\right)dT\right)\right]\frac{\lambda}{\eta}\left(\frac{E-\gamma}{\eta}\right)^{\lambda-1}exp\left[\left(\frac{E-\gamma}{\eta}\right)^{\lambda}\right]|\eta\in\tilde{\eta}[\alpha]\right\}$$

Let the fuzzy set of a trapezoidal fuzzy number be given by:
$\tilde{E} = \{a_1\ a_2\ a_3\ a_4\ a_1'\ a_2\ a_3 a_4'\}$
$\tilde{A} = \{b_1\ b_2\ b_3\ b_4\ b_1'\ b_2\ b_3\ b_4'\}$
$\tilde{\eta} = \{c_1\ c_2\ c_3\ c_4\ c_1'\ c_2\ c_3\ c_4'\}$
$\tilde{\lambda} = \{d_1\ d_2\ d_3\ d_4\ d_1'\ d_2\ d_3\ d_4'\}$

and define its membership $\xi_{\tilde{E}}, \xi_{\tilde{A}}, \xi_{\tilde{\eta}}, \xi_{\tilde{\lambda}}$ and non-membership $\phi_{\tilde{E}}, \phi_{\tilde{A}}, \phi_{\tilde{\eta}}, \phi_{\tilde{\lambda}}$ in the following manner:

$$\xi_{\tilde{E}} = \begin{cases} \dfrac{E-a_1}{a_2-a_1}, & a_1 \le E \le a_2 \\ 1, & a_2 \le E \le a_3 \\ \dfrac{a_4-E}{a_4-a_3}, & a_3 \le E \le a_4 \\ 0, & \text{otherwise} \end{cases} \qquad \xi_{\tilde{A}} = \begin{cases} \dfrac{A-b_1}{b_2-b_1}, & b_1 \le A \le b_2 \\ 1, & b_2 \le A \le b_3 \\ \dfrac{b_4-A}{b_4-b_3}, & b_3 \le A \le b_4 \\ 0, & \text{otherwise} \end{cases}$$

$$\xi_{\tilde{\eta}} = \begin{cases} \frac{\eta-c_1}{c_2-c_1}, & c_1 \le \eta \le c_2 \\ 1, & c_2 \le \eta \le c_3 \\ \frac{c_4-\eta}{c_4-c_3}, & c_3 \le \eta \le c_4 \\ 1, & \text{otherwise} \end{cases} \qquad \xi_{\tilde{\lambda}} = \begin{cases} \frac{\lambda-c_1}{c_2-c_1}, & d_1 \le \lambda \le d_2 \\ 1, & d_2 \le \lambda \le d_3 \\ \frac{c_4-\lambda}{c_4-c_3}, & d_3 \le \lambda \le d_4 \\ 1, & \text{otherwise} \end{cases}$$

$$\phi_{\tilde{E}} = \begin{cases} \dfrac{E-a_1'}{a_2-a_1'}, & a_1' \le E \le a_2 \\ 0, & a_2 \le E \le a_3 \\ \dfrac{a_4'-E}{a_4'-a_3}, & a_3 \le E \le a_4' \\ 1, & \text{otherwise} \end{cases} \qquad \phi_{\tilde{A}} = \begin{cases} \dfrac{A-b_1'}{b_2-b_1'}, & b_1' \le k_0 \le b_2 \\ 0, & b_2 \le k_0 \le b_3 \\ \dfrac{b_4'-A}{b_4'-b_3}, & b_3 \le k_0 \le b_4' \\ 1, & \text{otherwise} \end{cases}$$

$$\phi_{\tilde{\eta}} = \begin{cases} \frac{\sigma^2-c_1'}{c_2-c_1}, & c_1' \le \sigma \le c_2 \\ 0, & c_2 \le \sigma \le c_3 \\ \frac{c_4'-\sigma^2}{c_4'-c_3}, & c_3 \le \sigma \le c_4' \\ 1, & \text{otherwise} \end{cases} \qquad \phi_{\tilde{\lambda}} = \begin{cases} \frac{\lambda-d_1}{d_2-d_1'}, & d_1' \le \lambda \le d_2 \\ 0, & d_2 \le \lambda \le d_3 \\ \frac{d_4'-\lambda}{d_4'-d_3}, & d_3 \le \lambda \le d_4' \\ 1, & \text{otherwise} \end{cases}$$

The α-cut of the above functions is obtained as follow:

$$\tilde{E}[\alpha] = \{[a_1+\alpha(a_2-a_1), a_4-\alpha(a_4-a_3)][a_1'+\alpha(a_2-a_1), a_4'-\alpha(a_4'-a_3)]\}$$

$$\tilde{A}[\alpha] = \{[b_1+\alpha(b_2-b_1), b_4-\alpha(b_4-b_3)][b_1'+\alpha(b_2-b_1'), b_4'-\alpha(b_4'-b_3)]\}$$

$$\tilde{\eta}[\alpha] = \{[c_1+\alpha(c_2-c_1), c_4-\alpha(c_4-c_3)][c_1'+\alpha(c_2-c_1'), c_4'-\alpha(c_4'-c_3)]\}$$

$$\tilde{\lambda}[\alpha] = \{[d_1+\alpha(d_2-d_1), d_4-\alpha(d_4-d_3)][d_1'+\alpha(d_2-d_1'), d_4'-\alpha(d_4'-d_3)]\}$$

3. Application and Computational Methodology

A pine needle sample is used to evaluate thermal behavior and simulate the same results with the help of fuzzy set theory. To attain the pyrolysis conditions, nitrogen is considered as a purge and protective gas to ward off the ingression of pollutants. Before initializing the experiment, the furnace space is purged to eliminate the remaining oxygen. The volumetric rate of nitrogen of 200 mL min^{-1} is used to remove the product gases. Thermocouple type 'R' is used to measure the furnace temperature. The experiments were performed using a thermogravimetric analyzer (SII 6300 EXSTAR). A sample of 10.54 mg of pine needles is heated in a crucible pan of alumina at a heating rate of 10 °C min^{-1}. To prevent the buoyancy effect, correction measurements are used. For fuzzification of the kinetic parameters, an algorithm is designed by using MATLAB software.

Elemental composition and calorific value of the pine needle sample are shown in **Table 1**. These were computed with the help of a bomb calorimeter at a constant volume.

Table 1: Chemical composition of pine needles

Biomass Type	C	H	N	O	V.M*	H.H.V**	S	Ash
Pine Needle	53.64	5.36	0.62	33.92	68.4	20.8	0.20	2.1

*- Volatile Matter

**-Higher Heating Value

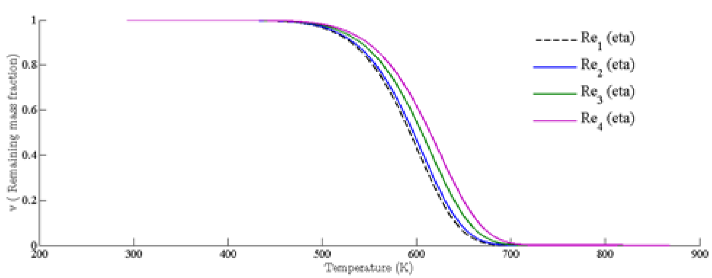

Figure 1: The effect of fuzzified scale parameter (η) of the fuzzy Weibull distribution on the numerical solution of the DAEM.

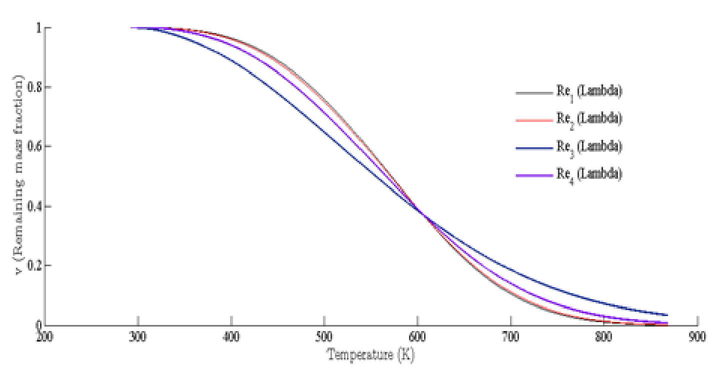

Figure 2: The effect of fuzzified shape parameter (λ) of the fuzzy Weibull distribution on the numerical solution of the DAEM.

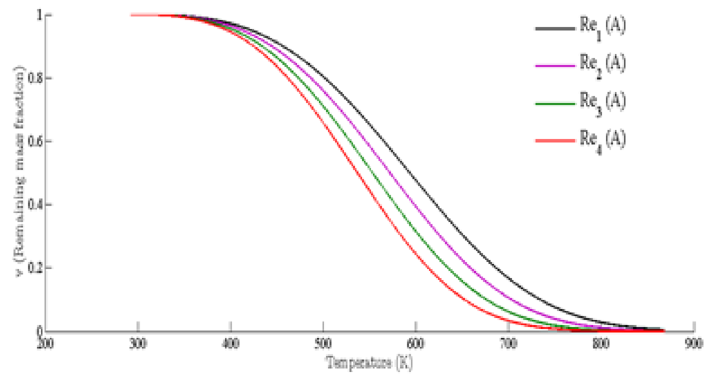

Figure 3: The influence of the fuzzy set of the frequency factor (A) on the numerical solution of the DAEM.

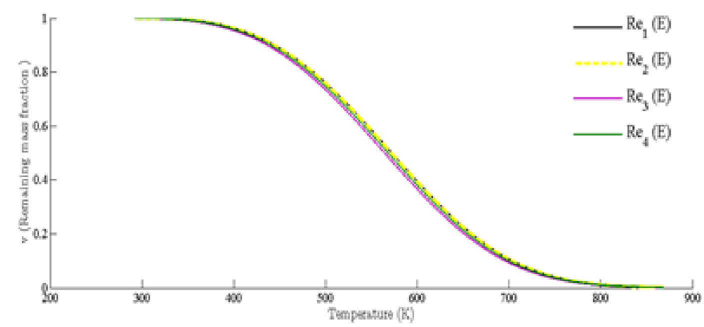

Figure 4: The effect of fuzzified upper limit of 'dE' integral (E) on the numerical solution of the DAEM.

4. Numerical Illustration

Assume the distribution parameters ($\tilde{\eta}$, $\tilde{\lambda}$) and kinetic parameters ($\tilde{A}$, $\tilde{E}$) are represented as the trapezoidal fuzzy number. The parametric values evaluated after fuzzy analysis are taken at *α=0*.

Let $\tilde{E}$(*kJ/mol*) = [(310, 311, 314, 316)], $\tilde{A}$ = [(10^{26}, 10^{28}, 10^{30}, 10^{31})], $\tilde{\lambda}$ =[(168, 170, 171, 175)] and $\tilde{\eta}$(*kJ/mol*) = [(3.1, 3.09, 3.06, 3.04)]. The influence of the fuzzy parameters on the final solution of the DAEM is expressed through members of the fuzzy sets Re_1 , Re_2 , Re_3 and Re_4. The Re_1 and Re_2 are the fuzzified members corresponding to membership function (ξ); while Re_3 and Re_4 correspond to the non-membership function. As the temperature increases, the fuzzy set shifts to the right and therefore the remaining mass fraction '*v*' shifts up for non-membership function.

After the fuzzification of the scale parameter (*η*) of the fuzzy Weibull distribution, the fuzzy set of membership and non-membership numbers are obtained, as shown in **Figure 1**. Had the nature of the scale parameters been crisp, all the members of the fuzzy set would have been merged to a single member and the behavior of v with respect to the distribution parameter would remain concealed. As the deviation of the membership function extended from the non-membership functions, the relative change in the remaining mass fraction v becomes narrow. The effect of the fuzzy shape parameter of the fuzzy Weibull distribution is illustrated in **Figure 2**. At the different members of the fuzzy set of the shape parameters λ, it can be seen that the v curves provide a good agreement with the thermo-analytical data. The influence of the fuzzified frequency factor on the final solution of the DAEM is shown in **Figure 3**. It can be seen that the remaining mass fraction curves shift to the left as the domain of the fuzzy set of A is extended. During the initial stage of pyrolysis, the remaining mass fraction remains in the neighborhood of one. The effect of the fuzziness of the upper limit of the "dE" integral on the numerical results is shown in **Figure 4**. The behavior of the remaining mass proportion is similar at the beginning and completion of the pyrolysis reactions, whereas there is a slight shift of inflexion points to the left with the increase in α-cut.

5. Conclusion

The fuzzy system has been successfully applied to the DAEM. Whenever randomness or fuzziness is encountered in the parametric values of any system, conventional methods become feasible. Thus, the fuzzy method was successfully invoked to overcome the complication of inaccuracy and imprecision in the kinetic parameters of biomass pyrolysis.

The benefits of categorizing the modelling parameters of biomass pyrolysis as member and non-member functions of fuzzy sets has overridden the demerits of the crisp-kinetic parameters which are indistinctive and unable to delineate the behavior of biomass pyrolysis due to Boolean logic. The fuzzy sets have handled the randomness or fuzziness to a desired level of accuracy, which is in turn helpful in making the analysis more realistic and practical. The influence of membership and non-membership functions on the remaining mass proportion is demonstrated in this study. Using this method, the fuzziness of the DAEM has been shown. The precise values of the kinetic parameters as well as the distribution parameters were obtained in the narrow width of the fuzzy subsets.

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