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Research article

Implementation of Fuzzy Sets in the Non-Isothermal Pyrolysis of Biomass

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Abstract

This paper proposes the study of the application of fuzzy logic on the relevant parameter of biomass pyrolysis. The frequency factor, the upper limit of 'dE', and the scale and shape parameters of the fuzzy Gamma distribution were fuzzified in order to estimate randomness and to accommodate realistic values. The distribution function, *f (E)*, of the activation energies was assumed to follow the fuzzy Gamma distribution. Thermo-analytical data was found experimentally with the help of TGA/DTG analysis. The approximated solution of the DAEM was obtained using the asymptotic approach.

1. Introduction

In various engineering systems, there is some level of imprecision regarding the assignment of values to some dependent parameters. While dealing with a mathematical model, we have to pay special attention to imprecision in data. The principle of incompatibility defines when the complexity of a system increases. Our aptitude to formulate precision and meaningful statements decreases up to

a threshold beyond which precision and significance become mutually exclusive characteristics **[1]**. It is not always feasible to believe the result obtained by a deterministic model is 100% correct. The main problem is whether replacing stubbornly imprecise data by fixed data will influence our investigation or not. If you substitute imprecise data with fixed values in a model, you will leave no choice to the model except churning out sometimes meaningless outcomes. Talking about false certainty is bad science and it can be dangerous if it stunts articulation of critical choices. Although probability theory claims to model decision making imprecision **[2]**, there are qualitatively different facets of indeterminacy which are not covered by probabilistic tools. In some situations where doubt arises about the exactness of a concept, correctness of statements and judgments have little to do with the occurrence of events, the backbone of probability theory. Such circumstances motivate many researchers to initiate investigation: how to incorporate non-stochastic impression into mathematical models **[3]**, **[4]**. Fuzzy set theory **[1]**, **[5]**, belief theory and evidence theory **[6]**-**[8]**, proceed along this line.

If we talk about fuzzy logic, which is a multi-value logic obtained from fuzzy set theory, interacting with human reasoning that ranges from 'almost certain' to the 'very unlikely'. As opposed to a classical propositional approach (true/ false), the membership value of fuzzy logic variables are not only 0 and 1 but can vary between 0 and 1 **[9]**. In order to understand it in much better way, we take an example of a 100 ml glass filled with 40 ml of water. Then we may consider two outcomes: Empty and full. The fuzzy set defines the meaning of both the outcomes. The glass can be defined to be 0.6 empty and 0.4 full. As the concept of emptiness is rather subjective, thus it depends on the observer or designer. The vagueness of emptiness and fullness can be tackled by using a membership function where the glass can be treated full for all values up to 50 ml. It is important to comprehend that fuzzy logic uses truth degree as a mathematical model of the vagueness phenomenon, whereas probability is a mathematical model of ignorance. A probabilistic model defines scalar variable which shows the fullness of the glass as a random event, and conditional distribution describes the probability that the glass is full to a given specific fullness level. This model, however, has no meaning unless some event occurs, i.e. after a few minutes, the glass can be half empty. Uncertainty and the time frame are the criteria of probability. An observer can randomly select the level of water in the glass, and achieve the condition as a distribution over other deterministic observers. Consequently, probability and fuzziness are not common but are two different concepts. They seem similar on the surface, as they use the same interval of real numbers [0, 1]. Many researchers have used the aforementioned facts in the mathematical modeling and analysis of engineering problems **[10]**, **[11]**.

In the present study, the aim is to implement a mathematical model of the vagueness on the relevant parameters of biomass pyrolysis. The crisp values of the kinetics as well as distribution parameters are fuzzified. The distributed activation energy model (DAEM) is approximated using asymptotic expansion. Note that the results of this article are used:

- To estimate the realistic value of kinetic parameters of loose biomass using the Gamma distribution,
- To identify the interval of variation (degree of truthiness in linguistic reasoning) of relevant parameters for biomass pyrolysis, and
- To see effect of fuzzification on the numerical solution of the DAEM.

2. Notations and Assumptions

Ew	Step size width (kJ/mol)
Es	Central Value (kJ/mol)
Y(x)	Lambert function
~	Symbol denotes fuzzy variable
H(x)	Heaviside step function
θ	Heating rate (°C/min.)
T	Temperature (°C)
R	Universal Gas constant (kJ/mol-K)
DExp	Double exponential term
f(E)	Distribution function
λ	Shape parameter
η	Scale parameter (kJ/mol)

3. Preliminaries

3.1 Fuzzy sets

The concept of fuzzy set theory was originally given by Zadeh **[12]** based on the assumption that the degree of membership is equal to one minus the degree of non-membership. In practical situations an object may or may not be a member of set A to a certain degree. In other words, some doubt about the degree of belongingness may exist. The idea of fuzzy sets is in tune with human representation of reality, as it is more about nuances than clear-cut definitions. Some related philosophical issues ranging from the ontological level to the application level via the epistemological level may be found elsewhere **[13]**.

In a fuzzy set, the degree of membership of an element is expressed by any real number from 0 to 1 rather than the limiting extremes. More formally, a fuzzy set of a set $A \neq \phi$ is characterized by a membership function ξ: A →[0, 1].

3.2 Membership and non-membership functions

It can be noted that the activation energy, frequency factor and variance are not clearly defined and are fuzzy, so are replaced with fuzzy numbers as:

$$\tilde{E} = \{a_1, a_2, a_3, a_4, a_1', a_2, a_{3,} a_4'\},$$

$$\tilde{A} = \{b_1, b_2, b_3, b_4, b_1', b_2, b_3, b_4'\},$$

$$\tilde{\eta} = \{c_1, c_2, c_3, c_4, c_1', c_2, c_{3,} c_4'\},$$

$$\tilde{\lambda} = \{d_1, d_2, d_3, d_4, d_1', d_2, d_3, d_4'\}$$

and membership $\xi_{\tilde{E}}, \xi_{\tilde{A}}, \xi_{\tilde{\eta}}, \xi_{\tilde{\lambda}}$ and non-membership $\phi_{\tilde{E}}, \phi_{\tilde{A}}, \phi_{\tilde{\eta}}, \phi_{\tilde{\lambda}}$

are defined in the following manner:

$$\xi_{\tilde{E}} = \begin{cases} \frac{E-a_1}{a_2-a_1}, & a_1 \le E \le a_2 \\ 1, & a_2 \le E \le a_3 \\ \frac{a_4-E}{a_4-a_3}, & a_3 \le E \le a_4 \\ 0, & \text{otherwise} \end{cases} \quad \xi_{\tilde{A}} = \begin{cases} \frac{A-b_1}{b_2-b_1}, & b_1 \le A \le b_2 \\ 1, & b_2 \le A \le b_3 \\ \frac{b_4-A}{b_4-b_3}, & b_3 \le A \le b_4 \\ 0, & \text{otherwise} \end{cases}$$

$$\xi_{\tilde{\eta}} = \begin{cases} \frac{\eta-c_1}{c_2-c_1}, & c_1 \le \eta \le c_2 \\ 1, & c_2 \le \eta \le c_3 \\ \frac{c_4-\eta}{c_4-c_3}, & c_3 \le \eta \le c_4 \\ 1, & \text{otherwise} \end{cases} \quad \xi_{\tilde{\lambda}} = \begin{cases} \frac{\lambda-c_1}{c_2-c_1}, & d_1 \le \lambda \le d_2 \\ 1, & d_2 \le \lambda \le d_3 \\ \frac{c_4-\lambda}{c_4-c_3}, & d_3 \le \lambda \le d_4 \\ 1, & \text{otherwise} \end{cases}$$

$$\phi_{\tilde{E}} = \begin{cases} \frac{E-a_1'}{a_2-a_1'}, & a_1' \le E \le a_2 \\ 0, & a_2 \le E \le a_3 \\ \frac{a_4'-E}{a_4'-a_3}, & a_3 \le E \le a_4' \\ 1, \text{otherwise} \end{cases} \quad \phi_{\tilde{A}} = \begin{cases} \frac{A-b_1'}{b_2-b_1'}, b_1' \le k_0 \le b_2 \\ 0, & b_2 \le k_0 \le b_3 \\ \frac{b_4'-A}{b_4'-b_3}, & b_3 \le k_0 \le b_4' \\ 1, & \text{otherwise} \end{cases}$$

$$\phi_{\tilde{\eta}} = \begin{cases} \frac{\sigma^2-c_1'}{c_2-c_1}, & c_1' \le \sigma \le c_2 \\ 0, & c_2 \le \sigma \le c_3 \\ \frac{c_4'-\sigma^2}{c_4'-c_3}, & c_3 \le \sigma \le c_4' \\ 1, & \text{otherwise} \end{cases} \quad \phi_{\tilde{\lambda}} = \begin{cases} \frac{\lambda-d_1}{d_2-d_1'}, & d_1' \le \lambda \le d_2 \\ 0, & d_2 \le \lambda \le d_3 \\ \frac{d_4'-\lambda}{d_4'-d_3}, & d_3 \le \lambda \le d_4' \\ 1, & \text{otherwise} \end{cases}$$

The α-cut of the above functions is obtained as follows:

$$\phi_{\tilde{E}} = \begin{cases} \frac{E-a_1'}{a_2-a_1'}, & a_1' \le E \le a_2 \\ 0, & a_2 \le E \le a_3 \\ \frac{a_4'-E}{a_4'-a_3}, & a_3 \le E \le a_4' \\ 1, \text{otherwise} \end{cases} \quad \phi_{\tilde{A}} = \begin{cases} \frac{A-b_1'}{b_2-b_1'}, b_1' \le k_0 \le b_2 \\ 0, & b_2 \le k_0 \le b_3 \\ \frac{b_4'-A}{b_4'-b_3}, & b_3 \le k_0 \le b_4' \\ 1, & \text{otherwise} \end{cases}$$

$$\phi_{\tilde{\eta}} = \begin{cases} \frac{\sigma^2-c_1'}{c_2-c_1}, & c_1' \le \sigma \le c_2 \\ 0, & c_2 \le \sigma \le c_3 \\ \frac{c_4'-\sigma^2}{c_4'-c_3}, & c_3 \le \sigma \le c_4' \\ 1, & \text{otherwise} \end{cases} \quad \phi_{\tilde{\lambda}} = \begin{cases} \frac{\lambda-d_1}{d_2-d_1'}, & d_1' \le \lambda \le d_2 \\ 0, & d_2 \le \lambda \le d_3 \\ \frac{d_4'-\lambda}{d_4'-d_3}, & d_3 \le \lambda \le d_4' \\ 1, & \text{otherwise} \end{cases}$$

4. Material and Methods

4.1 Distributed Activation Energy Model

The Distributed Activation Energy Model or Multiple Reaction Model assumes that many decomposition reactions take place simultaneously. It can also be understood as a summation of an unlimited number of parallel single step decomposition reactions, where each reaction has the following form:

$$\frac{dv_i}{dt} = A_{0i} \exp\left(-\frac{E_i}{RT}\right)(v^*_i - v_i) \quad \textbf{(1)}$$

where subscript i means one of several constituents , v_i is the total release mass o i^{th} constituent, t is time, A_{0i} is the frequency factor, E_i is the activation energy, R is the gas constant and T is the absolute temperature.

If the number of decomposition reactions involved is numerous, it can be assumed that the activation energies of these reactions are distributed, and the reactions can be expressed as a function of the activation energy.

$$dv^* = v^* f(E)\, dE \quad (2)$$

The right-hand side of equation (2) expresses the fraction of maximum mass loss v^* in the given interval of activation energy. Usually, $f(E)$ is taken to be a Gaussian distribution. However, with the selection of an appropriate distribution function for the molecular activation energies, it is advantageous to select an asymmetric distribution function for the molecular activation energies. The distribution of activation energy is considered to be a Gamma distribution function, over a symmetric function, i.e. Gaussian. Moreover, the Gamma distribution is mathematically flexible.

The DAEM equation for the non-isothermal time-dependent temperature regime can be derived by combining equations (1) and (2) using the Gamma distribution and it is given as:

$$v = \int_0^\infty \left[exp\left(\int_{T_0}^{T} \frac{-A}{\theta} exp\left(\frac{-E}{RT}\right) dT\right)\right] f(E) dE \quad (3)$$

where θ (°C/min) is the heating rate, E is activation energy and A is frequency factor (s^{-1}). The value of the frequency factor (A) is assumed to be constant for every decomposition reaction with various activation energies **[14]**. The value of the pre-exponential factor can also be expressed as a function of activation energy or temperature. The initial distribution of the activation energies can be expressed as:

$$v = \int_0^\infty \left[exp\left(\int_{T_0}^{T} \frac{-A}{\theta} exp\left(\frac{-E}{RT}\right) dT\right)\right] f(E) dE \quad (4)$$

The mean and the variance of the distribution are given by equations (5) and (6), respectively.

$$E_0 = \lambda\eta \quad (5)$$

$$\sigma^2 = \eta\lambda^2 \quad (6)$$

4.2 Asymptotic expansion

The integrand in equation (3) consists of two parts. The first part (DExp) of equation (3) depends on time through the range of temperatures experienced experimentally by the biomass samples. The second part is invariant to time and depends on the distribution of volatiles in the sample. The behavior of the temperature-dependent part is considered first, and approximations that are derived are useful for solving the double term. Only the ramping temperature history is investigated, together with the Gamma distribution of the volatiles.

Approximations to the double exponential are considered, where T varies linearly with time and has a constant slope, indicating the heating rate (θ) of the sample. E can take any positive value. The approach considered here is similar to that of Niksa and Lau **[15]**, but uses more systematic and more accurate approximation.

$$DExp = exp\left(-\int_0^t Ae^{-\frac{E}{RT}} dt\right)$$

For approximation of the double exponential term, the first step is to suppose the typical values of dependent parameters and functions. The frequency factors (*A*) are typically in the range of A~10^{10} - 10^{13} s^{-1}, whereas the domain of activation energies lies between 100 and 300 kJ/mol. The temperature dependence, however, is considered according to the specified experimental requirement. It varies from biomass to biomass, but typically the range of temperature varies as follows:

$$1000\ ^{\circ}C \leq T \leq 2000\ ^{\circ}C$$

Note: The DAEM model is also valid to combustion-related problems, where the extensive range of temperatures is used, thus it i useful to extrapolate the simplification mode in the higher specified regime of temperatures, as mentioned above.

In order to demonstrate the asymptotic technique, the ramping profile of temperature has been considered as follows:

$$T = \theta t,$$

If the temperature is assumed to ramp linearly, the double exponential term (DExp) becomes:

$$DExp = \exp\left(-\int_0^t Ae^{-\frac{E}{R\theta t}}\,dt\right)$$

The integral in the exponent is approximated using the conventional Laplace approach, where the parameter $\frac{E}{R\theta t}$ is assumed to be large and hence the dominant contribution from the integral is when temperature is near its maximum.

$$\text{Also, } \exp\left(-\int_0^t Ae^{-\frac{E}{R\theta t}}\,dt\right) \sim exp\left(\frac{-AR\theta t^2}{E}e^{-\frac{E}{R\theta t}}\right) \text{as} \frac{E}{R\theta t} \to \infty \quad (7)$$

Thus, equation (7) can be written in the form:

$$exp\left(\frac{-AR\theta t^2}{E}e^{-\frac{E}{R\theta t}}\right) \sim exp\left(-exp\left(\frac{E_s - E}{E_w}\right)\right)$$

where the function switches rapidly from zero to one with the increase in activation energy *E*, over a range of step sizes E_w around the central value E_s ,which can be approximated as follows:

Assuming,

$$g(E) = \left(\frac{E_s - E}{E_w}\right)$$

equation (7) can be rewritten as:

$$exp\left(-exp\big(g(E)\big)\right)$$

where

$$g(E) \equiv -\frac{E}{R\theta t} + ln\left(\frac{AR\theta t^2}{E}\right)$$

Expand *g (E)* with the help of the Taylor series around E_s

$$g(E) \sim g(E_s) + (E - E_s)g'(E_s) + \frac{(E - E_s)^2}{2!}g''(E_s) + \cdots \quad (8)$$

Using equation (8) and the predefined function $g(E_s)$, E_s and E_w are chosen such that

$$g(E_s) = 0 \text{ and } g'(E_s) = \frac{-1}{E_w}$$

After solving these, we have:

$$E_s = R\theta t Y(At) \text{ and } E_w = \left(\frac{R\theta t E_s}{R\theta t + E_s}\right)$$

where *Y(x)* is Lambert W function defined to be one of the real roots of the equation:

$$Y\,e^{Y} = x$$

Approximation for the small and large values of x (corresponding to short and long times) **[6]**.

$$Y \sim x - x^2,\ x \ll 1,$$

and

$$Y \sim ln\left(\frac{x}{ln\left(\frac{x}{lnx}\right)}\right), \qquad x \gg 1.$$

DExp has been varied as a smooth step-function rising rapidly (for the large values of *tA*) from zero to one in a range of activation energies of the step width E_w around the central value $E=E_s$, where both E_s and E_w vary with time. In equation (3), DExp is multiplied by the initial distribution *f(E)*. The initial distribution is assumed to be centered around a value E_0 and has a width designated by *σ*. The distribution can be either wide or narrow. When the distribution function *f(E)* is relatively wide in comparison to the width of DExp, the total integrand behaves similarly to an initial distribution *f(E)*. However, as time proceeds, it is progressively shifted from the left by the step-like DExp. The location of the maximum of the total integrand can move significantly, and the shape becomes skewed.

From equations (3) and (7), the remaining mass fraction equation can be expressed as:

$$v = \int_0^{\infty} exp\left(-exp\left(\frac{E_s - E}{E_w}\right)\right)\frac{E^{(\lambda-1)}e^{-\frac{E}{\eta}}}{\eta^{\lambda}\Gamma(\lambda)}\,dE$$

Let

$$h(E) = -exp\left(\frac{E_s - E}{E_w}\right) - \frac{E}{\eta}, \text{ then}$$

$$v = \int_0^{\infty} exp\big(h(E)\big)\frac{E^{(\lambda-1)}}{\eta^{\lambda}\Gamma(\lambda)}\,dE$$

where E_s and E_w are functions of *t*, as mentioned earlier.

Energy is now rescaled as $y = \frac{E}{E_0}$, so that the problem becomes:

$$v = \frac{\alpha}{\Gamma(\lambda)} \int_0^{\infty} y^{(\lambda-1)} exp(h(y))\ dy \tag{9}$$

$$h(y) = -exp\left(\frac{y_s - y}{y_w}\right) - \lambda y \tag{10}$$

where the constant parameter $\alpha = \left(\frac{\sigma^2}{E_0}\right)^{\lambda}$. Note that in practice $\alpha << 1$

Time is also rescaled as $\tau = At$
For linear ramping temperature $T = \theta t$

$$y_s = \frac{R\theta\tau Y(\tau)}{AE_0}, \qquad y_w = \frac{y_s}{(1 + Y(\tau))}$$

where *Y* is the lambert *W* function.

Note: The ramping temperature can be generalized to the case of non-zero initial temperature T_0 by simply replacing t with $t + \frac{T_0}{\theta}$ everywhere, else analysis will be changed.

Approximations to equation (10) are studied by considering the initial distribution, centered around $y = 1$ with width $\sqrt[2\lambda]{\alpha}$, while DExp jumps from zero to one around $y = y_s$ with a width y_w.

An initial distribution much wider than DExp is considered. In this limit, as previously discussed, DExp jumps from zero to one near $y = y_s$ in a manner that has previously been approximated by the step function **[16]**-**[19]**.

In order to apply this, the limit $y_w \sqrt[2\lambda]{\alpha} \ll 1$ is taken

$$H(y - y_s) = \begin{cases} 0, & y < y_s \\ 1, & y \geq y_s \end{cases}$$

Equation (9) can be rewritten in the form:

$$v = \frac{\alpha}{\Gamma(\lambda)} \int_0^{\infty} \left(\exp\left(-exp\left(\frac{y_s - y}{y_w}\right)\right) - H(y - y_s)\right) y^{(\lambda-1)} \exp\left(-\left(\sqrt{\lambda y}\right)^2\right)\ dy + \frac{\alpha}{\sigma^{2\lambda}\Gamma(\lambda)}\, \Gamma\left(\lambda, \left(\sqrt{\lambda y}\right)^2\right)$$

where $\Gamma(\lambda, \sigma^2 y_s)$ is the upper incomplete Gamma Function

The second integral in this equation is a cumulative distribution function, and therefore easily computable. Many previous simplifications (the step-function approximations) used this same term and neglected the first integral. The first integral term is the initial distribution multiplied by a function that is very small everywhere except in a neighborhood of size y_w around the point $y = y_s$. This integrand can, therefore, be approximated by expanding the initial distribution term with the help of a Taylor series about $y = y_s$

$$\text{Let } Z(y) = y^{(\lambda-1)} \exp\left(-\left(\sqrt{\lambda y}\right)^2\right)$$

$$Z(y) \sim Z(y_s) + (y - y_s)Z'(y_s) + \frac{(y - y_s)^2}{2!} Z''(y_s) + \frac{(y - y_s)^3}{3!} Z'''(y_s)$$

$$\sim exp\left(-\left(\sqrt{\lambda y}\right)^2\right)\left[y_s^{(\lambda-1)} - (y - y_s) y_s^{(\lambda-2)} (-\lambda + \lambda y_s + 1) + \frac{(y - y_s)^2}{2} y_s^{(\lambda-3)} \left(\lambda^2 y_s^2 + 2(1 - \lambda)\lambda y_s + (\lambda^2 - 3\lambda + 2)\right) - \frac{(y - y_s)^3}{6} y_s^{(\lambda-4)} (\lambda^3 y_s^3 + 3(1 - \lambda)\lambda^2 y_s^2 + (3\lambda^2 - 9\lambda + 6)\lambda y_s - \lambda^3 + 6\lambda^2 - 11\lambda + 6)\right]$$

Substituting $\frac{y - y_s}{y_w} = x, dy = y_w dx$ in equation (9), we have:

$$\sim exp\left(-\left(\sqrt{\lambda y}\right)^2\right)\left[y_s^{(\lambda-1)} - y_w x y_s^{(\lambda-2)} (-\lambda + \lambda y_s + 1) + \frac{(y_w x)^2}{2} y_s^{(\lambda-3)} \left(\lambda^2 y_s^2 + 2(1 - \lambda)\lambda y_s + (\lambda^2 - 3\lambda + 2)\right) - \frac{(y_w x)^3}{6} y_s^{(\lambda-4)} (\lambda^3 y_s^3 + 3(1 - \lambda)\lambda^2 y_s^2 + (3\lambda^2 - 9\lambda + 6)\lambda y_s - \lambda^3 + 6\lambda^2 - 11\lambda + 6)\right]$$

$$v = \frac{\alpha}{\Gamma(\lambda)} \int_0^{\infty} \left(\exp(-exp(-x)) - H(x)\right) exp\left(-\left(\sqrt{\lambda y}\right)^2\right) \left[y_s^{(\lambda-1)} - y_w x y_s^{(\lambda-2)} (-\lambda + \lambda y_s + 1) + \frac{(y_w x)^2}{2} y_s^{(\lambda-3)} \left(\sigma^4 y_s^2 + 2(1 - \lambda)\lambda y_s + (\lambda^2 - 3\lambda + 2)\right) - \frac{(y_w x)^3}{6} y_s^{(\lambda-4)} (\lambda^3 y_s^3 + 3(1 - \lambda)\lambda^2 y_s^2 + (3\lambda^2 - 9\lambda + 6)\lambda y_s - \lambda^3 + 6\lambda^2 - 11\lambda + 6)\right] y_w dx + \frac{\alpha}{\sigma^{2\lambda}\Gamma(\lambda)}\, \Gamma\left(\lambda, \left(\sqrt{\lambda y}\right)^2\right)$$

or

$$v \sim \frac{\alpha}{\Gamma(\lambda)} exp\left(-\left(\sqrt{\lambda y}\right)^2\right) y_s^{(\lambda-1)} y_w \left[L_0 - \frac{y_w}{y_s} L_1(-\lambda + \lambda y_s + 1) + \frac{1}{2}\left(\frac{y_w}{y_s}\right)^2 L_2\left(\lambda^2 y_s^2 + 2(1 - \lambda)\lambda y_s + (\lambda^2 - 3\lambda + 2)\right) - \frac{1}{6}\left(\frac{y_w}{y_s}\right)^3 L_3(\lambda^3 y_s^3 + 3(1 - \lambda)\lambda^2 y_s^2 + (3\lambda^2 - 9\lambda + 6)\lambda y_s - \lambda^3 + 6\lambda^2 - 11\lambda + 6)\right] + \frac{\alpha}{\sigma^{2\lambda}\Gamma(\lambda)}\, \Gamma\left(\lambda, \left(\sqrt{\lambda y}\right)^2\right) \tag{11}$$

We know that

$$\frac{\Gamma\left(\lambda, \left(\sqrt{\lambda y}\right)^2\right)}{\Gamma(\lambda)} = 1 - P\left(\lambda, \left(\sqrt{\lambda y}\right)^2\right)$$

where $P(\lambda, \sigma^2 y_s) = \frac{\gamma\left(\lambda, \left(\sqrt{\lambda y}\right)^2\right)}{\Gamma(\lambda)}$ is the lower cumulative distribution for Gamma random variables, and $\gamma\left(\lambda, \left(\sqrt{\lambda y}\right)^2\right)$ is the lower incomplete Gamma function.

$L_0 \approx -0.5772,\ L_1 \approx -0.98906,\ L_2 \approx -1.81496, L_3 \approx -5.89037$

The remaining integral terms are evaluated by the expression:

$$L_n \equiv \int_{-\infty}^{\infty} x^n \left(e^{-e^{-x}} - H(x)\right)\ dx$$

4.3 Fuzzy Gamma distribution function

If the remaining mass fraction of biomass is modeled by the Gamma distribution, then

$$f(\tilde{E}) = \frac{\tilde{E}^{(\tilde{\lambda}-1)} e^{-\frac{\tilde{E}}{\tilde{\eta}}}}{\tilde{\eta}^{\tilde{\lambda}}\Gamma(\tilde{\lambda})}, E > 0, \lambda > 0, \eta > 0,$$

$$v = \int_0^{\infty} \left[exp\left(\int_{T_0}^{T} \frac{-\tilde{A}}{\theta} exp\left(\frac{-\tilde{E}}{RT}\right) dT\right)\right] \frac{\tilde{E}^{(\tilde{\lambda}-1)} e^{-\frac{\tilde{E}}{\tilde{\eta}}}}{\tilde{\eta}^{\tilde{\lambda}}\Gamma(\tilde{\lambda})} dE \tag{12}$$

The fuzzy probability of obtaining a value in the interval [*n, m*], is as

$$\tilde{P}(n \leq X \leq m)$$

$$\tilde{P}(n < v < m)[\alpha] = \left\{ \int_n^m \left[exp\left(\int_{T_0}^{T} \frac{-\tilde{A}}{\theta} exp\left(\frac{-\tilde{E}}{RT}\right) dT \right) \right] \frac{\tilde{E}^{(\tilde{\lambda}-1)} e^{-\frac{\tilde{E}}{\tilde{\eta}}}}{\tilde{\eta}^{\tilde{\lambda}}\Gamma(\tilde{\lambda})} dE \middle| E \in \tilde{E}[\alpha] \right\}$$

$$= \left[P^L[\alpha], P^U[\alpha]\right],$$

$$\tilde{P}(n < v < m)[\alpha] = \left\{ \int_n^m \left[exp\left(\int_{T_0}^{T} \frac{-\tilde{A}}{\theta} exp\left(\frac{-\tilde{E}}{RT}\right) dT \right) \right] \frac{\tilde{E}^{(\tilde{\lambda}-1)} e^{-\frac{\tilde{E}}{\tilde{\eta}}}}{\tilde{\eta}^{\tilde{\lambda}}\Gamma(\tilde{\lambda})} dE \middle| A \in \tilde{A}[\alpha] \right\}$$

$$= \left[P^L[\alpha], P^U[\alpha]\right]$$

$$\tilde{P}(n<v<m)[\alpha]=\left\{\int_n^m\left[exp\left(\int_{T_0}^{T}\frac{-\tilde{A}}{\theta}exp\left(\frac{-\tilde{E}}{RT}\right)dT\right)\right]\frac{\tilde{E}^{(\tilde{\lambda}-1)}e^{-\frac{\tilde{E}}{\tilde{\eta}}}}{\tilde{\eta}^{\tilde{\lambda}}\Gamma(\tilde{\lambda})}dE\middle|\lambda\in\tilde{\lambda}[\alpha]\right\}$$
$$=\left[P^L[\alpha],P^U[\alpha]\right],$$
$$\tilde{P}(n<v<m)[\alpha]=\left\{\int_n^m\left[exp\left(\int_{T_0}^{T}\frac{-\tilde{A}}{\theta}exp\left(\frac{-\tilde{E}}{RT}\right)dT\right)\right]\frac{\tilde{E}^{(\tilde{\lambda}-1)}e^{-\frac{\tilde{E}}{\tilde{\eta}}}}{\tilde{\eta}^{\tilde{\lambda}}\Gamma(\tilde{\lambda})}dE\middle|\eta\in\tilde{\eta}[\alpha]\right\}$$
$$=\left[P^L[\alpha],P^U[\alpha]\right]$$

for all α, where

$$P^L=min\left\{\frac{\tilde{E}^{(\tilde{\lambda}-1)}}{\tilde{\eta}^{\tilde{\lambda}}\Gamma(\tilde{\lambda})}e^{\left[\int_{T_0}^{T}\frac{-\tilde{A}}{\theta}exp\left(\frac{-\tilde{E}}{RT}\right)dT-\frac{\tilde{E}}{\tilde{\eta}}\right]}\middle|E\in\tilde{E}[\alpha]\right\},P^U=max\left\{\frac{\tilde{E}^{(\tilde{\lambda}-1)}}{\tilde{\eta}^{\tilde{\lambda}}\Gamma(\tilde{\lambda})}e^{\left[\int_{T_0}^{T}\frac{-\tilde{A}}{\theta}exp\left(\frac{-\tilde{E}}{RT}\right)dT-\frac{\tilde{E}}{\tilde{\eta}}\right]}\middle|E\in\tilde{E}[\alpha]\right\}$$

$$P^L=min\left\{\frac{\tilde{E}^{(\tilde{\lambda}-1)}}{\tilde{\eta}^{\tilde{\lambda}}\Gamma(\tilde{\lambda})}e^{\left[\int_{T_0}^{T}\frac{-\tilde{A}}{\theta}exp\left(\frac{-\tilde{E}}{RT}\right)dT-\frac{\tilde{E}}{\tilde{\eta}}\right]}\middle|A\in\tilde{A}[\alpha]\right\},P^U=max\left\{\frac{\tilde{E}^{(\tilde{\lambda}-1)}}{\tilde{\eta}^{\tilde{\lambda}}\Gamma(\tilde{\lambda})}e^{\left[\int_{T_0}^{T}\frac{-\tilde{A}}{\theta}exp\left(\frac{-\tilde{E}}{RT}\right)dT-\frac{\tilde{E}}{\tilde{\eta}}\right]}\middle|A\in\tilde{A}[\alpha]\right\}$$

$$P^L=min\left\{\frac{\tilde{E}^{(\tilde{\lambda}-1)}}{\tilde{\eta}^{\tilde{\lambda}}\Gamma(\tilde{\lambda})}e^{\left[\int_{T_0}^{T}\frac{-\tilde{A}}{\theta}exp\left(\frac{-\tilde{E}}{RT}\right)dT-\frac{\tilde{E}}{\tilde{\eta}}\right]}\middle|\lambda\in\tilde{\lambda}[\alpha]\right\},P^U=max\left\{\frac{\tilde{E}^{(\tilde{\lambda}-1)}}{\tilde{\eta}^{\tilde{\lambda}}\Gamma(\tilde{\lambda})}e^{\left[\int_{T_0}^{T}\frac{-\tilde{A}}{\theta}exp\left(\frac{-\tilde{E}}{RT}\right)dT-\frac{\tilde{E}}{\tilde{\eta}}\right]}\middle|\lambda\in\tilde{\lambda}[\alpha]\right\}$$

$$P^L=min\left\{\frac{\tilde{E}^{(\tilde{\lambda}-1)}}{\tilde{\eta}^{\tilde{\lambda}}\Gamma(\tilde{\lambda})}e^{\left[\int_{T_0}^{T}\frac{-\tilde{A}}{\theta}exp\left(\frac{-\tilde{E}}{RT}\right)dT-\frac{\tilde{E}}{\tilde{\eta}}\right]}\middle|\eta\in\tilde{\eta}[\alpha]\right\},P^U$$
$$=max\left\{\frac{\tilde{E}^{(\tilde{\lambda}-1)}}{\tilde{\eta}^{\tilde{\lambda}}\Gamma(\tilde{\lambda})}e^{\left[\int_{T_0}^{T}\frac{-\tilde{A}}{\theta}exp\left(\frac{-\tilde{E}}{RT}\right)dT-\frac{\tilde{E}}{\tilde{\eta}}\right]}\middle|\eta\in\tilde{\eta}[\alpha]\right\}$$

5. Application

For application purposes, the experiment was conducted for the non-isothermal pyrolysis of pine needles. It is to be noted that the result of this study was used in the fuzzification process to obtain the realistic and authentic values of the parameters. **Table 1** indicates the chemical composition which is also obtained with the help of CHNO-S analysis of pine needle samples.

Table 1: Chemical composition of pine needles

Biomass Type	C	H	N	O	V.M*	H.H.V**	S	Ash
Pine Needle	53.64	5.36	0.62	33.92	68.4	20.8	0.20	2.1

*Volatile matter **High heating value

6. Results and Discussion

6.1 Numerical illustration

Assuming that the initial distribution of the activation energy follows the Gamma distribution with fuzzy scale parameters $\tilde{\lambda},\tilde{\eta}$, $\tilde{A}$ and $\tilde{E}$ taken as the trapezoidal fuzzy number, the values of parameters obtained after fuzzification are taken at α= 0.

Let $\tilde{E}\,(kJmol^{-1})=[(12.28,15.329,15.829,16.32)]$, $\tilde{A}(s^{-1})[(59.73,53.658,46.2,33.54)]$, $\tilde{\lambda}(kJmol^{-1})=[(0.7,0.68,0.6,0.8)]$ and $\tilde{\eta}=[(3.6,3.55.3.4.3.2)]$.

The effect of fuzzified parameters with temperature on the numerical solution of the DAEM is depicted by means of fuzzified bands Re_1, Re_2, Re_3, Re_4. The Re_1 and Re_2 represent the fuzzified bands corresponding to the membership function; whereas Re_3 and Re_4 correspond to the non-membership function. After the fuzzy analysis of the relevant parameters of biomass pyrolysis, four bands were obtained. If the nature of the parameters had been crisp, all four fuzzified bands would have converged to a single band. At the beginning of the pyrolysis process, the remaining mass proportion must be close to 1. However, in **Figure 1**, it can be seen that the remaining mass proportion is less than one for the fuzzy set (12.28,15.329). In order to obtain the most realistic and closely proximate results, the fuzzy set of the upper limit of the 'dE' integral should belong to (12.28,15.329). The effect of the fuzzified frequency factor (*A*) is illustrated in **Figure 2**. According to these curves, the increase in the width of the fuzzy set causes *v* curves to move towards right. The effect of the fuzzified scale parameter on the numerical solution is depicted in **Figure 3**. It can be seen that the resulting mass fraction curve is shifted up the temperature scale with the increase in the size of the fuzzy set. The effect of the fuzzy scale parameter ($\tilde{\eta}$) is shown in **Figure 4**.

A slight increase in the width of the fuzzy set of $\tilde{\eta}$ reduces the slope of the resulting mass fraction curves. In addition to the present study, several other methodologies and conceptual studies have been carried out to evaluate kinetic parameters with the help of the model-fitting method, viz. DAEM. Cai and Lui **[20]** implemented Simpson's 1/3 rule to evaluate the DExp. It is worth mentioning here, however, that the dependence of estimated values relies strongly on the truncation error, arising due to the upper limit, the lower limit and the maximum value of the function, all of which lead to a drastic deviation from the realistic outcome. The other method used for this problem is the asymptotic technique. In the present approach, the asymptotic technique has been implemented in symmetric as well as asymmetric functions to find an accurate and realistic value of the function. To some extent truncation error has been reduced by the asymptotic approximation of DExp **[21]**-**[24]**, yet there is some problem attached to the approximated solution of the DAEM. Furthermore, the crisp nature of the solution has a degree of randomness and is sometimes unrealistic due to the likelihood of the existence of more than one solution for the same function. In 2017, Dhaundiyal and Tewari trifurcated biomass into its three major constituent parts, namely Cellulose, Hemicellulose and Lignin **[2]**. They derived the approximate solution for each of constitutes and then summed them. The solution obtained through this technique of separately analyzing each component worked effectively **[25]**. In 2016, Dhaundiyal and Singh **[26]** analyzed the symmetric function using fuzzy sets to estimate to what extent the same outcome will be valid, converting the result into a crisp set with the help of boundary limits and an alpha-cut. They succeeded in doing this, but the fact that the experimental data can be either asymmetric or symmetric was overlooked in their study.

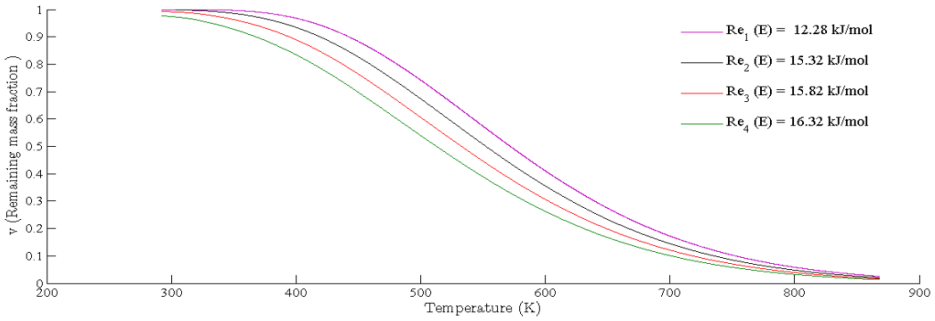

Figure 1: The effect of fuzzified upper limit of "dE" integral of the DAEM (E) (kJ mol^{-1}) on the numerical results

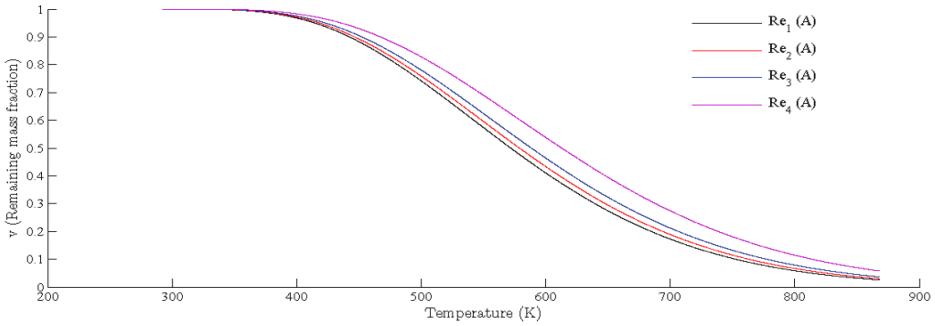

Figure 2: The effect of fuzzified upper limit of "dE" integral of the DAEM (E) (kJ mol^{-1}) on the numerical results

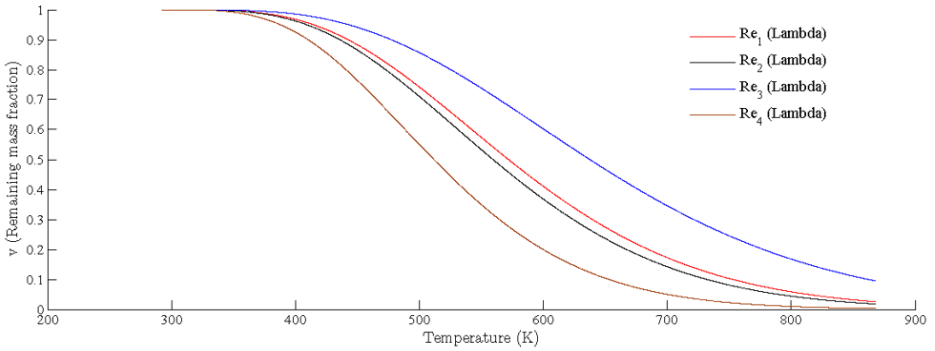

Figure 3: The effect of fuzzified upper limit of "dE" integral of the DAEM (E) (kJ mol^{-1}) on the numerical results

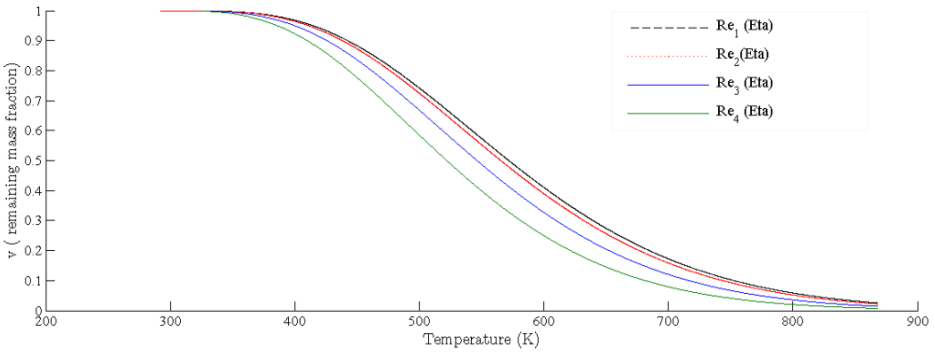

Figure 4: The effect of fuzzified upper limit of "dE" integral of the DAEM (E) (kJ mol^{-1}) on the numerical results

7. Conclusion

In this article the relevant parameters of the DAEM are considered to be fuzzy in nature in order to incorporate the realistic situation of the possible existence of uncertainty. Whenever, the distribution of activation energies, the frequency factor, and the parameters of the distribution function are vague and fuzzy, the conventional system is found to be unfeasible. Therefore, in order to overcome this complication, we have successfully implemented fuzzy theory. The use of fuzzified kinetic parameters in the DAEM has overcome the disadvantages of the crisp parameters, since the crisp values do not provide the practical approach to solving the kinetics problem. The fuzzy sets handled the uncertainty and vagueness to a certain desired level, which in turn helped in making our analysis more realistic and authentic. The membership and the non-membership function of the fuzzy sets were computed. Using the fuzzy Gamma distribution, the relevant parameters of biomass pyrolysis were approximated to some other realistic values. It can be seen that the membership function provides good agreement with the thermo-analytical data for a relatively large range of activation energies for which the DAEM closely approximates the experimental results.

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