# A Comparison of the Effectiveness of Using CRA-SIM vs. Direct Instruction to Teach Multiplication With Regrouping

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## Abstract

Mathematics learning standards across the United States include conceptual knowledge of numbers and use of modeling and explanation within computation. The concrete-representational-abstract (CRA) sequence and the Strategic Instruction Model (SIM) have been shown to be effective in increasing students' fluency in computation, including multiplication with regrouping, with a focus on conceptual knowledge. To date, the CRA-SIM multiplication research has not included a comparison to another method. The purpose of this study was to fill this gap by comparing CRA-SIM to Direct Instruction (DI). Twenty-nine elementary- and middle-level students with and without disabilities received either CRA-SIM or DI. While each program resulted in increased fluency in multiplication with regrouping, CRA-SIM led to slight increases in fluency compared to DI. Implications and program components that influenced these results will be discussed.

*Keywords:* multiplication, mathematics difficulties, CRA-SIM, Direct Instruction

Mathematics standards adopted by most states across the United States include mathematical practices intended to lead students toward effective mathematical thinking and application within real-world situations (Common Core State Standards Initiative [CCSSI], 2010). Mathematical practices, in turn, include identification of meaning within problems so that reasonable solutions can be developed. Evaluation of possible solutions requires deep knowledge and understanding of numbers and operations such that multiple pathways toward a solution can be generated and then judged for reasonableness (Barlow & Harmon, 2012).

In order for students to engage in these practices, instruction must emphasize conceptual understanding, perseverance in problem solving, as well as proficiency in procedural knowledge and fluency in operations (National Mathematics Advisory Panel, 2008). It is important for students to have conceptual knowledge because poor conceptual understanding often leads to confusion or error patterns as mathematics problem solving becomes more complex (Kroesbergen, van't Noordende, & Kolkman, 2014). Therefore, development of conceptual understanding should begin in the early grades and continue as more complex mathematical processes are introduced (Witzel, Ferguson, & Mink, 2012). Further, mathematics interventions for students who struggle should include modeling and emphasis on conceptual understanding as fluency develops (Dacey & Drew, 2012; Kiuhara & Witzel, 2014).

As operations become more complex, algorithms are included as one way to solve problems. Within the standards adopted by most states across the nation, students are required to show fluency in standard algorithms, including multiple-step processes associated with multiplication with regrouping and long division. When designing interventions that lead to fluency in using traditional algorithms, conceptual understanding as to how and why these algorithms work may be included.

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## Teaching Conceptual Understanding of Numbers and Operations

The use of the concrete-representational-abstract (CRA) instructional sequence is one way to build or reinforce conceptual understanding of numbers and operations. To assist students in learning and remembering procedural steps, CRA has been combined with the Strategic Instruction Model™ (SIM), a model in which students learn to systematically complete complex academic tasks through explicit instruction and programmed generalization with procedural knowledge acquired through a streamlined set of steps, usually in the form of a mnemonic device (Deshler & Schumaker, 1986).

The combination of CRA and SIM (CRA-SIM) works as follows. First, students learn to compute problems using concrete objects through explicit instruction (i.e., teacher demonstration, guided practice, and independent practice). Next, students solve problems by drawing representations of numbers, fading their dependence on physical objects. After having mastered computation using drawings, students learn a strategy for following algorithmic procedures in the form of a device that aids memory. This mnemonic strategy provides students with a scaffold from modeling problems with drawings to solving problems using numbers only. The goal of CRA-SIM is to develop computational fluency based on conceptual understanding of numbers and operations.

**CRA-SIM.** Mercer and Miller (1992) began the line of CRA-SIM research with the development of a curriculum called the *Strategic Math Series*, in which the researchers taught basic addition, subtraction, multiplication, and division using CRA-SIM. Within separate curriculum materials, students learned to perform basic operations using concrete objects, drawings, and a procedural mnemonic strategy called "DRAW" for solving problems using numbers only. The procedural steps were as follows: (a) discover the sign; (b) read the problem; (c) answer or draw and check; and (d) write the answer. Mercer and Miller (1992) found that CRA-SIM was more effective than traditional basal mathematics curriculum materials.

In a later study, Miller and Mercer (1993) investigated the effects of CRA-SIM for elementary students with specific learning disabilities (SLD), and obtained similar results with regard to effectiveness. Additional studies have demonstrated positive effects using CRA-SIM to teach basic operations as well as more advanced skills such as fractions and algebraic equations (Harris, Miller, & Mercer, 1995; Morin & Miller, 1998), and integers, fractions, and algebra (Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Witzel, Mercer, & Miller, 2003).

The most recent CRA-SIM research focused on computation involving regrouping, first with addition and subtraction and then with multiplication. For example, Flores (2009, 2010) taught subtraction with regrouping to elementary students experiencing failure in mathematics using explicit instruction combining CRA methods and the DRAW mnemonic strategy. Results of this multiple-probe-across-students study showed a functional relation between CRA-SIM and subtraction with regrouping across all students.

In another study, Miller and Kaffar (2011) compared CRA-SIM to traditional basal materials, teaching addition with regrouping to students with learning difficulties in mathematics. Whereas the comparison group received 16 lessons from a traditional second-grade basal series, the CRA-SIM group received 16 lessons, including explicit instruction using the CRA sequence and a procedural mnemonic strategy (Read the problem, Examine the ones column, Note ones in the ones column, Address the tens column, Mark tens in the tens column, Examine and note hundreds and exit with a quick check; RENAME). The researchers extended the regrouping research by having students use place-value mats to organize base-ten blocks and drawings during the concrete and representational stages of instruction. After solving addition with regrouping problems with objects and drawings, students learned the mnemonic strategy, RENAME, as they transitioned to problem solving using numbers only in the abstract phase. Compared to the basal group, students in the CRA-SIM group demonstrated significant gains in addition with regrouping.

Using a multiple-probe-across-students design, Mancl, Miller, and Kennedy (2012) extended the line of CRA-SIM subtraction with regrouping research to include students with SLD. The students solved problems using base-ten blocks and a place-value mat at the concrete level of instruction. During the representational phase, they learned to solve problems using pictures and a place-value mat. The researchers taught the RENAME strategy between the representational and abstract phases of instruction; finally, students progressed to problem solving using numbers only. The results indicated a functional relation between CRA-SIM and accuracy in subtraction.

Flores, Hinton, and Strozier (2014) taught subtraction with regrouping and multiplication with regrouping using a multiple-probe-across-behaviors design. Students receiving tier three intervention within a response to intervention prevention framework (Vaughn & Fuchs, 2003) learned subtraction with regrouping in the tens place, subtraction with regrouping in the tens and hundreds places, and multiplication with regrouping using one-digit and two-digit multipliers (bottom number). The algorithms were taught using base-ten blocks during the concrete phase and drawings during the representational phase. The students learned the RENAME strategy and applied it when solving problems during the abstract phase. A functional relationship was found between CRA-SIM and students' regrouping performance across subtraction and multiplication.

In an extension of this line of CRA multiplication research, Flores, Schweck, and Hinton (2014) included elementary students with SLD. Focusing solely on problems with two-digit multipliers, their study added place-value mats, which students used to organize base-ten blocks and drawings during the concrete and representational phases. Using a multiple-probe-across-students design, results showed a functional relation between CRA-SIM and multiplication with regrouping performance.

Finally, Flores and Franklin (2014) explored CRA research on methods implemented by practitioners instead of a researcher or graduate assistant, with findings that indicated CRA improves student performance for multiplication of two-digit numbers.

**Direct Instruction.** As indicated, the research supporting CRA-SIM for teaching multiplication with regrouping has demonstrated promising results for small groups of students. However, to date, no study has compared CRA-SIM to another method as a way of showing its effectiveness compared to other evidence-based practices with a focus on multiplication with regrouping.

One form of instruction classified as an evidence-validated instructional method for students who struggle with mathematic concepts is Direct Instruction (DI) (Gersten et al., 2009). DI has been found to be effective in teaching mathematics for at-risk students and students with disabilities (Przychodzin, Marchand-Martella, Martella, & Azim, 2004). DI programs are comprehensive and establish long-term goals that address strands, or skill sets embedded in larger concepts. For example, *Corrective Mathematics* addresses skill sets of solving for facts, place value, operations, and word problems within the larger idea of what addition, subtraction, multiplication, and division imply (Przychodzin et al., 2004).

When applying DI, the following procedures are used (Watkins & Slocum, 2004). First instruction organizes central concepts and strategies to ensure efficient student learning. Second, clear and systematic methods of teacher communication are implemented to decrease misunderstanding. Third, instruction includes structured verbal exchanges between teachers and students, which increase student engagement, progress monitoring, and repeated practice. Fourth, instruction strategically integrates skills to ensure efficient learning and understanding. Fifth, instructional concepts are arranged in strands, in which learning develops throughout the program while continually reviewing and generalizing information.

Similar to CRA-SIM, researchers have demonstrated positive effects of using DI to teach operations as well as more advanced mathematics such as fractions (Adams & Engelmann, 1996). Studies examining fraction instruction have included students with and without SLD in grades seven and higher (Flores & Kaylor, 2007; Kelly, Gersten, & Carnine, 1990; Scarlato & Burr, 2002). Since several skill sets are addressed in DI research, it is difficult to hone in on specific operations; nevertheless, studies have addressed embedded skills such as multiplication. Most DI research makes comparisons to basal programs used within the school systems and involves solving word problems using mathematic operations.

Darch, Carnine, and Gersten (1984) taught word problems with addition, subtraction, multiplication, and division operations to students in the fourth grade who demonstrated a deficit in addition, subtraction, multiplication, or division, and in solving for word problems. Seventy-three participants were randomly assigned to one of four groups: (a) DI method with fixed amount of practice, (b) basal instruction with fixed amount of practice, (c) DI method with extended practice, and (d) basal instruction with extended practice. The students who received DI outperformed peers who received basal instruction in solving word problems. The DI method with extended practice group scored the highest of all four groups.

In a later study, Wilson and Sindelar (1991) compared the effectiveness of using a DI and a basal program to teach addition and subtraction word problems. The study included 62 students with SLD in mathematics within nine elementary schools. Participants were able to solve addition and subtraction facts with at least 80% accuracy and read at least at a 1.5 grade level. Students were randomly assigned to three groups: (a) DI with word problem types taught individually, (b) DI with word problem types integrated, and a (c) basal program with word problems taught individually. Students who received DI made substantial increases in solving mathematical word problems involving addition and subtraction compared to students who received the basal program instruction.

Other researchers (Woodward & Brown, 2006) compared instructional approaches using the lens of standards-based curriculum. Specifically, Woodward and Brown (2006) examined a DI curriculum and a curriculum that was not explicit, but followed the National Teachers of Mathematics Standards 2000 guidelines. Participants, who were in the sixth grade, included students with and without disabilities, and were recommended by teachers for intensive, remedial instruction. The students had difficulty solving whole-number problems using the four basic operations (i.e., addition, subtraction, multiplication, and division) and place value. Results of the study showed that students who received DI instruction significantly outperformed the comparison group on posttest measures of the four basic operations and place value.

The purpose of the current study was to extend the CRA-SIM literature by comparing its effects to DI, an established, evidence-validated, and explicit instructional method.

# Method

## Participants

Participants were 29 students in the United States in grades four through seven enrolled in remedial summer programs based on their lack of response to instruction and their performance on the annual state assessment, indicating that they had not made adequate progress toward mastering state standards. Fifteen of the students had identified disabilities and were eligible for special education services under the categories of specific learning disabilities (7), emotional behavioral disorders (3), autism spectrum disorders (1), and other health impairments (4).

The criteria for student participation were as follows: (a) parental permission to participate in research, and (b) meeting placement guidelines in the DI *Corrective Mathematics Multiplication* (Engelmann, & Carnine, 2005) program, which meant students had to demonstrate proficiency in basic multiplication, addition with regrouping, and multiplication involving one-digit numbers. The students were matched as pairs by assessment performance,

Table 1
*Student Demographic Information by Group*

| Demographic Information | | DI Group | CRA-SIM Group |
|---|---|---|---|
| Gender | Male | 10 | 9 |
| | Female | 5 | 5 |
| Grade | Grade 4 | 6 | 8 |
| | Grade 5 | 5 | 5 |
| | Grade 6 | 3 | 1 |
| | Grade 7 | 0 | 1 |
| Ethnicity | White | 11 | 13 |
| | African American | 2 | 1 |
| | Latino/a | 1 | 1 |
| Disability | Specific Learning Disability | 4 | 3 |
| | Emotional Behavioral Disorder | 2 | 1 |
| | Autism Spectrum Disorder | 1 | 0 |
| | Other Health Impairment | 1 | 3 |
| | None | 6 | 8 |
| Computation Achievement[a] | | Standard Score = 82<br>Range = 72-98 | Standard Score = 81<br>Range = 68-99 |

[a]Standard score from Operations Subtest, *KeyMath Diagnostic Assessment* (Connolly, 2007).

disability category when possible, ethnicity, and grade. The members of each pair were randomly assigned to either the CRA-SIM (15 students) or DI (14 students) groups through a coin toss. Statistical analysis showed there were no differences between the groups. Student characteristics by group are presented in Table 1.

## Setting

The study was conducted in remedial summer intervention programs at a combined elementary and middle school. The annual programs are a collaborative effort between a university and a local school district in a small Midwestern city. Students attended the program for four and one half hours per day, four days per week for six weeks. District teachers provided general instruction in reading, written expression, and mathematics. In addition, university graduate students pulled children out for up to two periods per day to provide more intensive mathematics interventions. The graduate students were certified teachers enrolled in a special education licensure program for specific learning disabilities; five teachers had recently completed a bachelor's degree and initial certification with less than one year of teaching experience, and one teacher had 17 years of general education teaching experience.

Instruction occurred in separate classroom settings for 50 minutes each day. For both DI and CRA-SIM, the students received small-group instruction with four to six students in each group. Across groups, students were seated at tables or desks surrounding the teacher, who provided instruction using a white board and materials appropriate for each intervention.

## Materials

**Direct Instruction materials.** The teacher materials consisted of the *Corrective Mathematics Multiplication* presentation book (Engelmann & Carnine, 2005). The teachers used the script within the book and the white board to implement the program. The student materials consisted of the *Corrective Mathematics Multiplication* student workbook. The student lesson materials involved a page with multiple sections in which students completed short activities related to multiplication and multiplication with regrouping procedures; the students practiced multiplication facts, mental computation, application of place value, addition with regrouping, word problems, and multiplication with regrouping within each lesson. The materials also included a section in which students recorded points that could be earned for completion and accuracy within each lesson section.

Although it was not part of the curriculum, the students in the DI group also completed a learning contract prior to instruction, whereby they agreed to work hard to learn multiplication, and, in turn, their teachers agreed to work hard to provide instruction. The contract was added to the program because CRA-SIM includes a contract, and the researchers wanted equivalent student motivation within each group.

**CRA-SIM materials.** The teacher materials included (a) an instructional manual describing procedures and teacher behaviors for each lesson, including suggested scripts and answer keys; (b) a set of oversized magnetic base-ten blocks to be used for teacher demonstration; and (c) a reproduction of the place-value mat projected on the white board and Smart Board™ to organize magnetic blocks or draw representations.

Each student used the following materials: (a) learning contract signifying a commitment to learn a new way of multiplying; (b) a set of base-ten bocks made of foam; (c) a laminated multiplication mat that was used to organize base-ten blocks, learning sheets for each lesson, and a progress chart that was used to monitor daily progress during guided and independent practice. The learning sheets were divided into three sections: demonstration, guided practice, and independent practice. Within each section, problems were written with both words and mathematical symbols (e.g., There are 32 classrooms and each has 24 students. How many students in all? __ groups of __, 32 x 24). The problems within learning sheets 1 through 12 consisted of only multiplication problems. Beginning with lesson 13, instruction included discrimination between multiplication problems and addition or subtraction problems. Learning sheets for lessons 14-16 consisted of words without written problems requiring that students determine the appropriate operation as well as computation. Maintenance lessons began with lesson 17, which did not have demonstration, just guided and independent practice. Finally, lessons 18-20 involved only independent practice. In addition to learning sheets, student materials included progress charts, which were used to record accuracy of computation for each lesson.

**Assessment materials.** Assessment materials were two timed assessments developed by the first author. Probes consisted of sheets of paper with 20 problems requiring multiplication with regrouping. Both probes included similar problems, but the problems were not presented in the same order to ensure students would not memorize answer patterns from pretest to posttest.

Reliability of the researcher-developed probes was assessed prior to the study; results from the internal consistency test revealed Cronbach's alpha coefficient of $r$ = .83 for all probe items. Timed assessments were used since previous CRA research used fluency as a dependent measure.

## Assessment Procedures

Students completed timed probes before and after the DI or CRA-SIM interventions, as follows. The teachers gave each student a probe and told them that they would be taking a timed test and were to begin when a timer started. Students were also told to answer as many problems as possible until they were told to stop and the timer sounded. After these instructions, the teachers told the students to begin, and started a timer set for two minutes. When the timer sounded, the teachers told students to put their pencils down and collected the completed probes. The probes were analyzed by counting the number of correct digits written below the answer line, meaning the numbers used in calculating the problem as well as the final answer.

## Instructional Procedures

**Difference between DI and CRA-SIM procedures.** Both DI and CRA-SIM are explicit instructional methods, but differ in several ways. DI requires more frequent choral responding than CRA-SIM, which affords students more opportunities to practice correct responses when solving operations with multiplication with regrouping. DI also offers more opportunities for students to practice with repetition solving problems using the procedural components of the standard algorithm. CRA-SIM, on the other hand, places heavy emphasis on conceptual understanding by making models and drawings of quantities, and demonstrating the operation as students solve problems. Once students have solving problems in a manner that requires multiple representations of the numbers and operations (e.g., base-ten blocks, drawings, and numbers), they are allowed to solve problems using numbers only, with a focus on the procedures of the standard algorithm.

**DI.** The procedures for the DI group were followed, as prescribed by the program (Engelmann & Carnine, 2005). Each instructional period lasted 50 minutes; the size of the instructional group influenced the lesson length since individual turns and responses required more time. The students sat at tables or desks around the teacher. The teachers used the program script to present instruction. Instruction began with lesson 28, the entry point into the program for students who have mastered basic multiplication and need instruction in regrouping. Seventeen lessons that involved multiplication with regrouping (lessons 28-44) were implemented. Lessons included multiple skills that involved review of multiplication facts, reading and writing numbers, multiplication with single- and multi-digit multipliers, and word problems involving addition, subtraction, and multiplication.

A typical lesson went as follows. The first part was rehearsal of basic multiplication facts. Students completed addition with regrouping. Another section provided instruction in place value and reading numbers (e.g., 2,006 read as "two thousand six"). Here students completed word problems that involved addition, subtraction, and multiplication.

The section of the lesson devoted to multiplication with regrouping procedures provided students with tasks that were progressively more complex. Multiplication problems were written using grids that separated numbers by place value, and small boxes were written above the problem to be used for regrouping. The students were presented with a partially completed multiplication problem that had two-digit multipliers (bottom number).

Instruction began with the teacher's verbal description of how to multiply the numbers in the multiplicand (top number) by the number in the ones place of the multiplier (bottom number). Next, the program presented students with a problem written with grids, regrouping boxes, and completed computation using the number in the ones place of the multiplier. The students computed the rest of the problem using the number in the tens place of the multiplier and added the results to arrive at the final answer. Next, the program presented students with problems written with grids and regrouping boxes, but students computed the whole problem. Finally, the grids were removed, and students computed problems that included regrouping boxes. A visual representation of problem presentation for each program is presented in Figure 1.

Instruction involved activities requiring both oral and written responses using the student workbooks. Throughout each lesson, the program includes a point system whereby students earn points for completing lesson components correctly. The teachers implemented this reinforcement system across all lessons. Students were asked to respond to questions or engage in tasks as a group and individually. The teachers used signals to prompt group responses. The teachers followed the prescribed error-correction procedures for the following errors: responses prior to the teacher's signal, late responses, failure to complete tasks, and incorrect responses. Correction procedures included modeling the correct response,

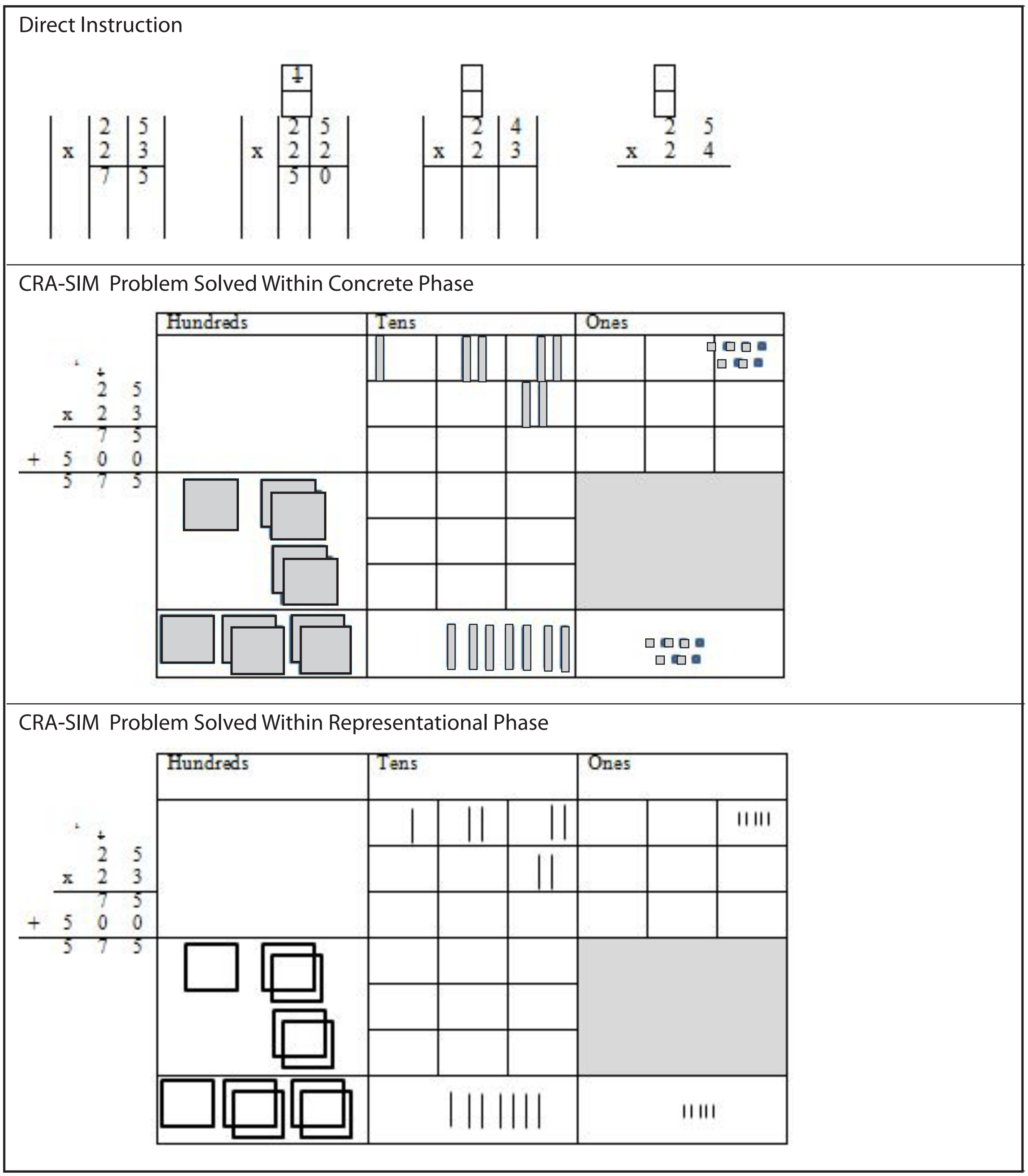


*Figure 1*. Visual presentation of instructional aids within each program.

leading the students in the correct response, and asking students to respond independently.

**CRA-SIM instruction.** For CRA-SIM, the teachers implemented instruction according to the intervention manual. The manual includes suggested scripts for each lesson intended to provide guidance but not to be read word for word. Each lesson consists of five parts: the advance organizer, demonstration, guided practice, independent practice, and post-organizer. During the advance organizer, the teachers provided the students with an overview of the lesson activities and stated expectations for student behavior. During the demonstration, the teachers showed the students how to solve problems through physical as well as mental processes by thinking aloud. Students were asked to participate by responding to questions related to prior knowledge or repeating words or phrases used by the teacher.

Guided practice involved problem solving by both the teacher and the students. The teachers completed problems with the students' assistance. That is, the students told the teachers how to complete each step of the problem and, when necessary, the teachers provided verbal prompts. Independent practice involved student completion of problems without the teachers' assistance. During this stage, the teachers monitored students' work and provided feedback when errors occurred, but did not complete portions of problems or prompt students during their work. The lesson ended with a post-organizer, in which the teachers briefly reviewed the lesson activities and provided group feedback.

The teachers measured students' progress using the number of correct digits written within the last problem completed during guided practice as well as the independent practice problems. There was no timing component to these portions of the lesson, but number of digits correct was used rather than the number of problems correct because it is a more sensitive measure of improvement. Lesson mastery was defined as 80% of digits correct. The students tracked their own progress after each lesson by marking their percent of digits correct on a chart.

The intervention was divided across 20 lessons, 17 of which were completed within this study. Prior to the first lesson, students took a pretest, discussed their performance with the teacher, and made a commitment to learning a new way of solving multiplication problems. The first five lessons involved solving multiplication problems with regrouping using base-ten blocks and a multiplication mat; instruction began with one-digit multipliers in lesson 1, and became more complicated with each lesson until problems included two-digit multipliers in lesson 5. After the advance organizer, the teachers demonstrated problems using large base-ten block magnets and the multiplication mat on a white board, large enough for students to see. The teachers read the written problem, emphasizing that there were groups of items/people and each containing the same amount and that this could be solved through multiplication. The teachers set out the multiplicand (top number) on the mat using base-ten blocks and used the digit in the ones place of the multiplier (bottom number) to make groups. They told students about a rule, "if there are ten or more, go next door." If the groups of ones combined to make 10 or more, the ones blocks were exchanged for tens blocks and placed in the tens' place of the mat. The teachers wrote a small number (crutch number) over the digit in the tens' place of the written problem to indicate regrouping. The teachers marked the remaining ones on the written problem and moved to the digit in the tens' place of the multiplicand.

The teachers made groups of tens using the digit in the ones' place of the multiplier. When combining the groups, they added the crutch number previously added to the tens' place during regrouping. Upon adding the number, the teachers crossed out the crutch number, telling the students that they did not want that number to confuse them later. The teachers applied the "ten or more" rule by exchanging 10 tens blocks for hundreds, if necessary. They marked the written problem. The teachers told the students that they were finished multiplying by the bottom number in the ones' place. They crossed out the digit in the ones' place of the multiplier and wrote a zero in the ones' place underneath the first row of answers, emphasizing that they were multiplying by tens. Then they multiplied the digit in the ones' place of the multiplicand by the digit in the tens' place of the multiplier. They made groups of tens and put them on the mat. They applied the "ten or more" rule if necessary and marked the problem. Then the teachers told the students that they were working the hundreds' place. The teachers showed the students the problem (e.g., 30x20, 30 groups of 20) and demonstrated that this resulted in six hundred using the blocks (30 groups of 20 made 60 tens blocks, which were combined into 6 hundreds blocks). The teacher told the students that 30 groups of 20 is the same as 3 groups of 2 hundreds.

After the teachers multiplied the multiplicand by both numbers in the multiplier using blocks and completed the written problem, they combined the blocks within each row of answers, beginning with the ones column. After combining blocks in each column, the teachers marked the written problem. Finally, the teachers checked the students' work by comparing the blocks with the written answers. The teachers demonstrated three problems. Guided practice involved completion of three problems through cooperation with students and assistance with verbal prompts. Finally, the students completed three problems independently.

Lessons 6 through 10 involved the use of drawings to represent the problem on the multiplication mat rather than base-ten blocks. The procedures described above were followed again. Ones were drawn using horizontal lines (groups) with short vertical tallies (items in each group). Tens were drawn using long vertical lines. Hundreds were drawn using squares. Visual representations of the instructional aids used in each program were presented in Figure 1; CRA-SIM representations show a completed problem rather than the entire process of manipulating objects and drawings.

During lesson 11, the students learned the strategy for solving regrouping problems. The RENAME strategy was taught and practiced until the students could recite the steps upon seeing the mnemonic device. This lesson served as a bridge between problem solving using representations and problem solving using abstract symbols or numbers only. With lesson 12, students solved problems using the RENAME strategy without concrete or representational tools.

Lessons 13 through 16 involved solving problems at the abstract level as well as discriminating between multiplication and other operations. It also included learning an additional strategy: Find what you are solving for, Ask yourself, "What are the parts of the problem?," Set up the numbers, Tie down the sign (FAST). The students practiced FAST RENAME until they could recite each the steps of FAST RENAME upon presentation of the mnemonic device. Lessons 14 through 16 involved solving problems using numbers only. These lessons also included discrimination between word problems (addition, subtraction, or multiplication), identifying the appropriate operation as well as computation of the problem.

When demonstrating problem solving, the teacher emphasized analysis of the words within problems and thinking about the whole problem. For example, if groups of items were being joined together, were there multiple groups with the same amount in each (multiplication) or were groups of different sizes combined (addition)? The inclusion of different types of problems was similar to the DI lessons and ensured that students understood real-world application of multiplication rather than simply assuming that word problems within a multiplication lesson would require multiplication.

Lessons 17 through 20 were maintenance lessons. Lesson 17 involved guided practice with two problems, and the remainder of the lesson was independent practice. Lessons 18 through 20 involved only independent practice; however, the study ended with Lesson 17.

## Treatment Integrity

Treatment integrity was measured using checklists completed after the researchers watched both live and videos of instruction. Twenty-five percent of the lessons were observed, with one observation and simultaneous video recording each week so that observations were spread throughout the intervention. Using a checklist based on checklists used in previous DI and CRA-SIM research (Flores , Hinton, & Stonier, 2014a; Marchand-Martell et al., 1995), the observer indicated whether 15 teacher behaviors that were required throughout the lesson were present or not. Teacher behaviors included (a) organization; (b) accurate presentation of lesson using scripts (DI) or suggested scripts (CRA-SIM); (c) accurate use of program procedures; (d) accurate correction and feedback procedures; (e) accurate use of program materials; (f) appropriate pacing and affect such as smooth phrasing, eye contact with students, and engaging tone of voice; and (g) maintenance of progress monitoring (CRA-SIM) or point system (DI). One of the authors observed a lesson live, another observed the recordings and calculated interobserver agreement. Interobserver agreement for treatment integrity scoring was 98%.

The treatment integrity for the DI group was 96%, with scores ranging from 80% to 100%. The behaviors lacking were related to organization, correction procedures, and management of choral responses. The treatment integrity for the CRA-SIM group was 93%. The behaviors lacking here were related to organization, pacing, procedures, and teacher affect.

The researchers met with the teachers after poor ratings (i.e., poor ratings were defined as scores below 80% for not having materials ready, not modeling or solving problems with students; and displaying disinterest to students' reactions and cues) to provide feedback and practice; to ensure remediation, the researchers observed the subsequent lesson (this follow-up observation was not included in treatment integrity).

## Interscorer Reliability and Social Validity

The teachers and a researcher scored the assessment probes, calculating the number of correct digits written. A researcher compared scores and calculated interscorer reliability by dividing the number of agreements and the sum of agreements and disagreements. Interscorer reliability was 97%.

Social validity was measured using both teacher and student surveys. After the study, teachers and students completed a survey regarding their satisfaction with and the effectiveness of the multiplication interventions. Both surveys included questions about liking instruction, increasing their multiplication skills, and whether they would recommend the instruction type for other students. Specific questions were related to each type of instruction, such as the type of instructional materials (multiple skills in one lesson for DI or use of base-ten blocks and drawings for CRA-SIM), procedures such as frequent choral responding, and use of motivational systems such as earning points and using a progress chart.

**Students.** Survey questions were read aloud to the students, who completed them as a group without providing their names. The DI student survey results indicated that 93% of students increased their multiplication skills and liked the materials. Most (71%) students liked the fast pace of instruction, the point system, as well as the lesson format, which involved different types of activities in each lesson. The students' responses were more divided regarding choral responses; 57% reported that this was difficult.

The CRA-SIM student survey results indicated that all of the students in this group perceived that their multiplication skills increased. Most (93%) students reported that they liked the lesson format and would recommend it for other students. More than half (60%) of the students did not like using blocks and reported that manipulation of blocks with the place-value mat was difficult. However, 100% of the students liked solving problems using drawings on the place-value mat, and 93% reported that they would continue to use the RENAME strategy to solve problems. Most (93%) students liked the progress chart.

**Teachers.** The teachers all reported that the students were in need of a multiplication with regrouping intervention and that the programs were beneficial. The DI teachers agreed that the lesson design with multiple skills was effective and also liked the scripted lessons, but they were split in their responses about other components of the program. Two teachers liked the pacing, frequent questioning, and choral response, and felt DI was easy to implement. The other teacher did not like these features of the program, and reported that she would not use it again.

The three CRA teachers' responses were similar to each other. That is, the teachers reported that the three phases of instruction were beneficial for students, the program was easy to implement, and they would use it again. Both CRA-SIM teachers reported that their least favorite part of the program was management of materials at the concrete level.

# Results

Prior to the intervention, the researchers established that the DI and CRA-SIM groups were equivalent with regard to pretest performance and mathematics achievement, defined as standard scores on the Operations subtest of the *KeyMath-3* (Connolly, 2007). A one-way analysis of variance (ANOVA) was conducted, with the between-subjects factor being group (DI or CRA-SIM) and the within-subjects factors being the number of correct digits written on the pretest and the operations standard score on the *KeyMath-3*.

Results of the ANOVA indicated that there were no significant differences between groups with regard to pretest performance, $F(1, 27) = 0.10$, $p = 0.75$. Similarly, there were no differences between groups with regard to computation achievement, $F(1, 27) = 0.05$, $p = 0.82$. Therefore, both groups were equivalent. The means and standard deviations are presented in Table 2.

The researchers analyzed data for all students present for the pre- and posttest using a repeated-measures ANOVA. A two-way ANOVA was conducted with the between-subjects factor being group (DI or CRA-SIM) and the within-subjects factor or dependent variable being number of digits correct as written on timed probes before and after the intervention. The means and standard deviations for number of digits correct are presented in Table 2.

The results of the ANOVA indicated a significant change in student performance across groups, Wilks' $\Lambda = 0.495$, $F(1, 27) = 27.54$, $p < 0.000$, multivariate $\eta^2 = 0.51$. The results also revealed a significant effect for group, meaning that there was a difference between groups, Wilks' $\Lambda = 0.78$, $F(1, 27) = 7.61$, $p < 0.01$, multivariate $\eta^2 = 0.22$.

Table 2
*Means and Standard Deviations for the Two Groups*

| Analysis of Group Equivalence | | Mean | Standard Deviation |
|---|---|---|---|
| Pretest (correct digits written on 2-minute probe) | | | |
| | DI | 15.29 | 15.36 |
| | CRA-SIM | 17 | 13.83 |
| Standard Score on Operations Subtest *KeyMath-3* | | | |
| | DI | 82.14 | 6.92 |
| | CRA-SIM | 81.47 | 9.1 |

| Number of Digits Written Correctly on Timed Test | | Mean | Standard Deviation | Effect Size |
|---|---|---|---|---|
| DI Group | | | | |
| | Pretest | 15.29 | 3.9 | |
| | Posttest | 22.0 | 4.52 | |
| | | | | $\eta^2 = 0.22$ |
| CRA-SIM Group | | | | |
| | Pretest | 17 | 3.77 | |
| | Posttest | 38.6 | 4.52 | |

## Discussion

The purpose of the study was to compare CRA-SIM to another explicit evidence-validated practice, DI, in the form of *Corrective Mathematics Multiplication* (Engelmann & Carnine, 2005). Both instructional programs were implemented with fidelity by teachers in natural settings within a summer intervention program. The average fluency for both groups increased, and participation in either intervention resulted in moderate gains ($\eta^2 = 0.51$). While there was a larger increase for students within the CRA-SIM group, the effect size was minimal ($\eta^2 = 0.22$). For 12 of the students in the CRA-SIM group, their fluency score increased, and of those, 9 students demonstrated increases of more than 20 correct digits (range of 21-58). In the DI group, the fluency of 9 students increased, and of those, 2 demonstrated increases of 20 and 23 correct digits, respectively.

Three students in the CRA-SIM group and six students in the DI group did not demonstrate progress. The researchers informally compared the characteristics of these students to students whose fluency increased in an attempt to find a reason for this finding. The presence of disability did not differ since students with the same disabilities within their instructional groups showed increases from 15 to 39 digits. In addition, *KeyMath* scores for these students were similar; the lowest standard score within the group of non-responders was 69, but some students with *KeyMath* scores of 72 and 73 increased their fluency by 12 to 44 digits. Students who did not demonstrate progress were in grades four and five across both groups, but their same-age peers showed increases. Finally, three non-responding students from each group had five or more absences whereas students who demonstrated increases in fluency had one or fewer absences. As a result, exposure to instruction through attendance likely influenced student progress.

In addition, results indicated that (a) more than half (60%) of the students did not like using blocks and reported that manipulation of blocks with the place-value mat was difficult; (b) 100% of the students liked solving problems using drawings on the place-value mat; and (c) 93% reported that they would continue to use the RENAME strategy to solve problems. Base ten-blocks are used at the beginning of the intervention to represent quantities and operations in a concrete manner as students solve problems and write the numbers that correspond to the concrete representations. Perhaps it was difficult and cumbersome for students to manipulate and trade out blocks to regroup at the beginning of the intervention. Using base-ten blocks may also be viewed negatively by students, as a sign that they need help to solve the operation. When drawing representations or using RENAME to solve problems, students do not trade out quantities to regroup, which can make it obvious to others what they may know or not know. Instead, they draw out how the quantities were regrouped on their personal mat, or solve problems using RENAME and numbers only on their paper.

The students within the CRA-SIM group demonstrated learning, consistent with previous CRA-SIM research related to multiplication with regrouping (Flores & Franklin, 2014 ; Flores, Hinton, & Strozier, 2014; Flores, Schweck, & Hinton, 2014). The students' performance on probes was also consistent with previous research. Students' fluency did not show increasing trends until the end of the abstract phase. In the current study, the abstract phase was presented, but there was not time for further practice.

The results of this study are significant because CRA-SIM was compared to a research-validated explicit practice, whereas previous studies in this line of research have only documented student progress and response to CRA-SIM. Findings showed that CRA-SIM was as effective as DI and resulted in slight gains in fluency, or students' ability to solve problems accurately and quickly. This finding is significant because CRA-SIM was compared to a research-validated intervention program with an emphasis on explicit, teacher-directed instruction in very specific task-oriented skills systematically leading to fluent procedural knowledge. The DI program also included frequent choral responding, so students had more opportunities to practice correct responses. This is a different approach than CRA-SIM, which does not include these features; however, students in the CRA-SIM group made at least as much or more progress in fluency. The CRA-SIM intervention emphasizes conceptual understanding and mathematical thinking as students make models and representations while computing problems, and this approach resulted in equivalent gains in procedural fluency.

Both programs were explicit; the DI program was likely more explicit and provided more practice and repetition with regard to the procedural components of the algorithm using numbers only. However, the DI program lacked conceptual instruction using base-ten blocks, place-value mats, drawings, and a mnemonic for remembering the algorithmic procedures – components of CRA-SIM that may have increased students' conceptual understanding of the algorithmic procedures, influencing overall progress and progress toward mastery of the procedures.

Throughout instruction, video evidence from fidelity measures showed students in the CRA-SIM group answering questions and verbally describing the conceptual underpinnings of computation procedures. In addition, researchers engaged in informal interactions with students in CRA-SIM and DI groups regarding their approach to computation. The researchers asked students to talk aloud while solving a problem using numbers only. Students in the CRA-SIM group consistently referred to numbers based on their value (e.g., 20 x 40 with the answer being 8 hundred) whereas students in the DI group referred to these as the written numerals in the algorithm (e.g., 2 x 4). Although students in both groups wrote the number eight correctly in the hundreds place of the problem, the CRA-SIM group appeared to be more aware of the algorithm's meaning.

## Limitations

The current study is limited in terms of the length of implementation, approximately four and one half weeks of instruction with 17 lessons. Both programs include additional lessons to which students were not exposed. Additional time may have allowed for greater gains in students' multiplication performance. Thus, future research should be implemented without the time limits imposed by the summer programs within this study. Implementation during a traditional school year would allow for a natural implementation over the course of a grading period.

The study is also limited in terms of the number and representativeness of the subjects. As a result, future research should include larger groups of students receiving both interventions as well as students from different regions of the country.

## Conclusions and Implications

The current research provides additional evidence that CRA-SIM is an effective intervention with regard to multiplication with regrouping. This is the first study of multiplication with regrouping that involved groups of this size and comparison with an evidence-validated explicit practice. It is difficult to conclude that previous CRA-SIM multiplication-with-regrouping results would generalize due to the use of single-case methods and the small number of studies conducted. The current study strengthens this line of research and has greater implications for generalization since a larger number of children demonstrated progress, as much progress as an evidence-validated practice. CRA-SIM was implemented within a class period, four days per week – that is, within instructional limits similar to those within typical elementary and middle schools, especially when specific intervention periods are designated. In addition, the CRA-SIM and DI interventions were easily implemented based on teachers' reports.

This study provides data for informed decision making with regard to curricular choices for intervention. Both interventions were effective, although differing in the way conceptual knowledge is included. That is, the DI program addresses conceptual understanding through verbal description whereas the CRA-SIM program addresses conceptual understanding through student manipulation of base-ten blocks and drawings. CRA-SIM may be preferable for students who need extra scaffolding in their abstract thinking, provided through the concrete and representational phases of instruction. CRA-SIM also allows for student demonstration of mathematical practices (CCSSI, 2010) that call for modeling and explanation of problem solving. DI does not preclude this, but the physical process of modeling is not included. Overall, then, for the purposes of using interventions consistent with standards for mathematical practices, CRA-SIM may be preferable, but additional research is necessary to draw those conclusions.

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