Private Speech Use in Mathematics Problem Solving: A Review of Studies Comparing Children With and Without Mathematical Difficulties

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Abstract

Recent studies have concluded that children's development of private speech (private speech internalization) is related to and important for developing mathematical ability. In this article, we review a project consisting of studies exploring the cognitive factors that may underlie differences between the use of private speech by children with (MD) and without (MN) mathematical difficulties. The main issue of interest was whether private speech internalization is related to children's mathematical achievement, task-specific strategies, phonological awareness, and phonological memory, and whether any such relationships are modulated by age and mathematical achievement. The findings not only confirm that private speech internalization relate to mathematical achievement, they also highlight possible parallels between the contributions of strategies, phonological awareness, and phonological memory to subsequent mathematical achievement. Overall, the results seem to provide evidence for the hypothesis that mathematical achievement is causally related to phonological abilities – which underpin the internalization of private speech – rather than being directly related to the private speech internalization.

Keywords: mathematical difficulties, private speech, private speech internalization, strategy use, phonological awareness, phonological memory

Difficulties in Mathematics

Children with MD have a specific difficulty in mastering calculation despite adequate instruction and the absence of intellectual disability. It is estimated that 5-8% of students have a cognitive or neuropsychological deficit that interferes with calculation or word problem solving (Badian, 1983; Geary, 1993; Ostad, 1998).

To understand and address problems in mathematical cognition, it is essential to first identify the core difficulties that discriminate children with MD from their peers without such challenges. Previous theoretical models in the area of mathematical learning have focused on various sources of MD. Thus, difficulties in visuospatial representation of numerical information (Geary, 1993), impaired working memory (Geary, Hamson, & Hoard, 2000; Passolunghi & Siegel, 2004; Swanson & Sachse-Lee, 2001), use of developmentally immature calculation procedures (Jordan & Montani, 1997; Ostad, 1997; Ostad & Sorensen, 2007), difficulty learning basic arithmetical facts or retrieving them once they are learned (Geary, 1993; Jordan, Hanich, & Kaplan, 2003; Ostad, 1999; Swanson & Sachse-Lee, 2001), and poor conceptual understanding (Geary et al., 2000; Jordan et al., 2003) are all observed in young children with MD.

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Proposed mechanisms underlying these deficits range from fundamental deficits in potentially innate systems for processing number and magnitude (Butterworth, 2005) to specific deficits or delays in the mechanisms underlying mathematical ability (Hecht, Torgesen, Wagner, & Rashotte, 2001; Ostad, 2013; Swanson & Jerman, 2006); for further details, see Geary, Hoard, and Bailey (2012).

Children's Private Speech

Considerable research has investigated the cognitive abilities underpinning reading. For example, children's decoding reading ability is known to be highly correlated with phonological ability (Dolcos & Albarracin, 2014; Durand, Hulme, Larkin, & Snowling, 2005; Wagner, Torgesen, & Rashotte, 1994). Longitudinal correlation studies have made an important contribution to the emerging consensus that certain kinds of phonological processing ability; that is, the ability to use phonological or sound information in processing written and oral language is causally related to the development of reading skills and phonological impairments are a leading cause of reading disabilities (Ramus & Szenkovits, 2008; Snowling, 2001; Wagner et al., 1994).

More than 30 years of research on dyslexia has resulted in the identification of three main dimensions to the phonological deficit associated with the disorder: deficits in *phonological memory, phonological awareness, and phonological re- or decoding* (Hecht al., 2001; Kulak, 1993; Snowling, 2001). Of interest to the current study, it is possible that the poor mathematical ability of persons with MD is due to deficits in one or more of these dimensions (Hecht et al., 2001; Rasmussen & Bisanz , 2005; Wagner & Torgesen, 1987).

Vygotsky (1934/1986) hypothesised that the phenomenon of private speech (self-talk used by children in various situations that is not addressed to others) reflects children's potential for self-directed planning, guiding, and monitoring of personal goal-directed activity. From the Vygotskian perspective, and that, therefore, children's private speech can be considered an important intrapsychic tool for regulating thought and behaviour (Berk & Winsler, 1995; Winsler & Naglieri, 2003).

Development of private speech follows a developmentally typical course of increasingly sophisticated private speech categories, culminating in what we refer to as *private speech internalization*. Researchers have shown an overall ontogenetic pattern, whereby children's overt private speech is gradually replaced by partially internalized whispers, inaudible mutterings, and silent inner speech as they progress through elementary school (Berk, 1992; Diaz & Berk, 1992; Flavell, Green, Flavell, & Grossman, 1997; Kohlberg, Yaeger, & Hjertholm, 1968; Winsler & Naglieri, 2003).

Other aspects of children's private speech have also been explored, including its relationships with children's task performance and on-task behaviour (Winsler, Diaz, & Montero, 1997), task and setting influences on such speech (Winsler, Carlton, & Barry, 2000), its use among children with behaviour problems, learning impairment, or attention deficits (Berk & Landau, 1993), and task-related utterances to self during problem-solving (Winsler, Feder, Way, & Manfra, 2006).

The majority of recent studies have concluded that internalization of private speech is related to and important for the development of mathematical ability (Berk & Landau, 1993; Ostad, 2015; Winsler & Naglieri, 2003). Underlying this conclusion is the belief that arithmetic knowledge is stored in a sound-based, or phonological, form. Several variants of the phonological storage hypothesis have been proposed (Anderson-Day & Fernyhough, 2015; Cohen & Dehaene, 2000; McCloskey, 1992; Robinson, Menchetti, & Torgesen, 2002). It remains far from clear, however, whether private speech internalization itself plays any causal role in the development of mathematical ability.

The overall purpose of the project reported here was to add to our knowledge of the cognitive abilities underpinning private speech internalization. More specifically, whether private speech internalization is related to children's mathematical achievement, use of task-specific strategies, phonological awareness and phonological memory, and whether any such relationships are modulated by age and mathematical achievement.

The Present Project: Common Methods

Separate laboratory investigations were carried out to examine children's private speech, strategy use, phonological awareness, and phonological memory, respectively. For each child, the four investigations were finished within a seven-day period. The same children participated in all the investigations.

The study used a cross-sectional design. To ensure that the behaviour of interest was recorded under comparable conditions across all subjects and grade levels, the design included research procedures for individual observation developed for use outside the classroom. Order of administration of the investigations was counterbalanced across subjects at each of the grade levels included in the study.

Participants

Control for mathematical ability was based on the arithmetic subtest of the Wechsler Intelligence Scale for Children (WISC-R; Undheim, 1977) and a Standard Mathematics Performance Test (Hammervoll & Ostad, 1999). The latter appears to tap very basic number processing aptitudes, in which children with MD have deficits (Geary et al., 2012).

MD children, defined for the purposes of this series of studies as the less mathematically able children in their grades, were chosen from a population of children who met the following two criteria: (a) attained stanine scores in the range of 1-3 on the standard mathematics test (i.e., a score among the 23% weakest for the grade group) and (b) attained scaled scores in the range of 1-7 on the WISC-R subtest (i.e., a score among the 25% weakest for the relevant group).

To avoid false negatives, MN children, in turn, were defined as the most mathematically able children in all grades. These children were selected by asking general education classroom teachers to nominate the most mathematically able children in their grades they taught, and based on these nominations the researchers assigned a same-sex MN student as a match for each MD child. The final sample of MN children satisfied both of the following criteria: (a) attained stanine scores in the range 7-9 on the standard mathematics test (i.e., among the 23% strongest in the relevant grade group and (b) attained scaled scores in the range 12-19 on the WISC-R (i.e., among the strongest 25% for the relevant grade group).

The sample comprised 134 children (half MN children and half MD children) from five state schools in Norway. All children belonged to one of the following grade groups: Group 1: children in grades 2 and 3 ($n = 22 + 22$); Group 2: children in grades 4 and 5 ($n = 22 + 22$); and Group 3: children in grades 6 and 7 ($n = 23 + 23$). The respective mean ages for MD and MN children in each group were as follows, Group 1: 7.7 and 7.8 years; Group 2: 9.7 and 9.6 years; and Group 3: 11.7 and 11.8 years.

There were no statistically significant age differences between children with and without MD within the grades groups, $F(1,42) = 2.19$, $p > .05$ for Ggr1; $F(1,42) = 0.98$, $p > .05$ for Ggr2; $F(1,43) = 2.07$, $p > .05$ for Ggr3. Inspection of the MD children's records showed that the schools' support services had already identified all 67 children as being in need of a special programme of mathematics teaching. Children in special education classes were not included as members in the sample.

Instruments

Assessment tools of private speech were constructed from the 64 possible pairwise additive combinations of the integers 2 to 9, with tie problems (e.g., 2 + 2) excluded. The remaining problems, 56 single-digit addition problems in the form a + b, were divided into two halves (half a and half b). The two halves were counterbalanced so that all 56 problems were pair-wise matched (e.g., 9 + 8 and 8 + 9). Lots were drawn so that the one problem in each pair was randomly assigned to one half, with the other problem from the pair going to the other half. Only one of the two halves (half a or half b) was used in testing each child. The problems (28 problems in each of the two halves) were placed horizontally at the centre of 21 x 10 cm printed booklets, with one problem per sheet. Other equipment placed on the table where the children were tested included paper, pencil, and 40 cubes (sides measuring about 1.5 cm).

Observational Procedure

According to Fuson (1979), the presence of another person creates the challenge of separating social speech from private speech. To minimize communication with the researcher in the current study, the children were informed ahead of time that they had to answer all the problems without any reference to the researcher. At the start of the study, the children were told to solve problems that would be presented on the (28) pages in the"book." During the time interval when the child turned to the next page, the private speech category for solving the problem was recorded by the researcher using the classification scheme developed for the investigation.

The reliability of this structured data collection format was previously tested. Two master's degree students who had received extensive training in the research procedure developed for the study coded, independently of each other, private speech responses based on 100 randomly chosen addition problems. The overall inter-coder reliability (averaging the three private speech levels) for the broad category classification was 0.96. Inter-coder reliability for each private speech level was, as follows: Audible: 1.00; Inaudible: 0.97; Silence: 0.95.

Relationship Between Private Speech and Mathematical Achievement

There is no consensus on methods or standardised criteria for categorising units of private speech (Girbau, 2002). The categorisation system used in the studies reported here integrated techniques and ideas from earlier research (Berk, 1986; Girbau, 2002; Kohlberg et al., 1968). Specifically, we defined categories of private speech in relation to private speech internalization (Kohlberg et al., 1968; Vygotsky, 1987) as follows: (a) externalised verbal production by means of words or sounds, (b) externalised manifestation of private speech, and (c) silence (Berk & Landau, 1993; Girbau, 2002).

More specifically, we classified private speech as (a) Audible private speech (high, normal, or low) – private speech that was potentially intelligible to a listener and could be transcribed; (b) Inaudible private speech that was detectable by face-to-face observation but unintelligible to even a very nearby listener. This category is used to refer to externalised private speech that is not loud enough for a listener to discern any semantic content to verbalisation, including inaudible muttering, and may be evident from lip and tongue movements (Girbau, 2002). It seems to be widely accepted that private speech according to this classification unit represents an external manifestation of inner speech (Berk & Landau, 1993; Fuson, 1979; Kolberg et al., 1968). (c) Subvocal private speech; that is, private speech that is silent and not reflected in any external verbal production or in lip or tongue movements (Girbau, 2002; Vygotsky, 1987). In this report, subvocal speech was operationalised as a purely implicit, covert, mental process that was undetectable through face-to-face-observation. It seems reasonable to consider subvocal speech as representing the most internalised form of private speech.

These three categories of private speech were coded as audible private speech, inaudible private speech, and silence. The number of answers belonging to each of the three private speech categories formed the basis for a unitary measure of the children's private speech developmental level, also known as the *private speech internalization score* (the PSI score).

The three subcategories' relative contributions to the development of mathematical aptitude (Ostad, 2013; Ostad & Sorensen, 2007) were roughly estimated and scored in the proportion 0:1:2, respectively. (For example, the PSI score = 26 for the child with 11 answers belonging to category audible, 8 to inaudible, and 9 to in silence.) With reference to the private-speech literature (Berk, 1992; Diaz & Berk, 1992; Flavell et al., 1997; Kohlberg et al., 1968; Winsler & Naglieri, 2003), we suggest that the PSI score is at least a theoretically continuous variable.

Results

Table 1 presents descriptive information about the private speech use, expressed as PSI scores, for solving the corresponding number fact problems. The data (mean score and standard deviation), common for the three studies, are broken down initially by grade groups (grades 2-3, grades 4-5, and grades 6-7), and then further by achievement groups (MN children and MD children).

To examine whether or not the private speech development of the MD children showed different developmental patterns from those of the MN children, we performed a univariate ANOVA analysis, identifying achievement groups as between-subjects

Table 1

Means and Standard Deviations for Private Speech Internalisation Scores (PSI Score) by Grade and Mathematical Achievement Group

Mathematical Achievement Group	Ggr 1		Ggr 2		Ggr 3	
	M	*SD*	*M*	*SD*	*M*	*SD*
MD children	16.73	4.95	18.73	2.90	22.78	4.26
MN children	25.14	5.05	32.73	5.80	42.17	8.10

Note. Number of problems = 28. Ggr1= Grade group 1 (Grades 2-3); Ggr2 = Grade group 2 (Grades 4-5); Ggr3 = Grade group 3 (Grades 6-7); MD = Children with mathematical difficulties; MN = Children without mathematical difficulties.

factors and PSI score as the dependent factor. The test indicated that a child's achievement group had a significant effect: F (1, 132) = 115.94, $p < .01$ ($R^2 = .468$). Furthermore, ANOVAs were performed to determine if the PSI scores of MN and MD children differed in the three grade groups. The analysis revealed significant differences for all three groups: $F_{\text{Gr2-3}}$ (1, 42) = 31.03, $p < .01$ ($R^2 = .425$); $F_{\text{Gr4-5}}$ (1, 42) = 102.58, $p < .01$ ($R^2 = .710$); $F_{\text{Gr6-7}}$ (1, 44) = 103.23, $p < .01$ ($R^2 = .701$). (For further details about the results, see Ostad and Askeland, 2008.)

Relationship Between Private Speech and Strategy Use

Several studies have investigated the strategy use of individuals of various levels of arithmetical experience and ability, and in samples varying in age from very young children to adults (Carpenter & Moser, 1982; De Chambier & Zeiger, 2018; Dowker, 2005; Geary et al., 2000; Ostad, 1998; Siegler, 1988; Thevenot, Barrouillet, Castel, & Uittenhove, 2016). In particular, the developmental level of children's strategy use has been viewed as the basis of theoretical models of the pupils' mathematical ability (Siegler & Jenkins, 1989). For example, researchers have observed inter-individual variation in strategy used to solve a basic problem among children of a given age. There has also been interest in intra-individual variability.

Research over the past three decades has uncovered developmental changes in children's use of problem-solving strategies (Geary, 1993; Ostad, 1998; Siegler & Jenkins, 1989). MN children appear to progress from using overt strategies, to verbal counting, to fact retrieval (Carpenter & Moser, 1982; Siegler, 1988). By contrast, MD children seem to be characterised by (a) use of backup strategies only, (b) use of the most primary backup strategies, (c) use of a limited range of strategies, and (d) limited change in the use of strategies during the school years (Ostad, 1997). It is therefore not surprising that it has been hypothesised that differences in strategy use can be used to distinguish between individuals of different intellectual levels or aptitudes (Dowker, 2005; Ostad, 2000).

Assessment of Task-Specific Strategies

Several systems have been used to classify children's strategy use (Carpenter & Moser, 1982; Geary et al., 2000; Groen & Parkman, 1972; Ostad, 1999; Siegler & Shrager, 1984). In the present study, we used the three main aspects of the strategy use internalization process: backup strategies, decomposition strategies, and direct retrieval strategies (Geary et al., 2012; Ostad, 1997); for further details, see Ostad and Sorensen, 2007.)

As a basis for the data analysis, we introduce the notion of *strategy use internalization,* which refers to the movement through the typical chain of increasingly sophisticated task-specific strategy categories. We suggest that strategy use internalization might load along a continuum (analogue theoretical axis) from primary backup strategies at one extreme to direct retrieval strategies at the other.

The number of strategies belonging to each category (backup, decomposition, retrieval) formed the basis for a unitary measure of children's strategy developmental level, also called the *strategy-use internalization score* (STRAT score). The three subcategories' relative contributions to the development of mathematical aptitude were roughly estimated and scored in the proportion 0:1:2, respectively. For example, the STRAT score was 26 for the child with 11 answers belonging to backup, 8 to decomposition, and 9 to retrieval. With reference to the strategy use researchers (e.g., de Chambrier & Zesiger, 2018; Geary et al., 2012; Jordan et al., 2003; Ostad, 1997) have suggested that the STRAT score is at least a theoretically continuous variable.

The reliability of this structured collection format was previously tested. A preliminary examination of the constancy of children's strategy use related to the three main categories (i.e., backup, decomposition, and retrieval strategies) was conducted as follows: 10 children representing each of the six grade performance groups (a total of 60 children) were given a set of 15 addition problems, selected by drawing lots from the addition problems included in the study. These sets of problems were presented to the children twice with a time limit of 60 min. The frequency of using the same strategy category for solving each of the problems twice was expressed in terms of percentage of occurrence. In all the six groups, the percentage was between 97 and 100.

Table 2 presents descriptive data showing children's strategy use expressed as STRAT scores. The data (mean scores and standard deviations) are broken down initially by grade groups (Gr 2-3, Gr 4-5, and Gr 6-7), and then further by achievement groups (MN and MD children).

To examine whether or not the development of the strategy use of MD children showed different developmental patterns from those of MN children, we performed a univariate ANOVA identifying

Table 2

Means and Standard Deviations for Strategy Use Internalisation Scores (STRAT Score) by Grade and Mathematical Achievement Group

Mathematical Achievement Group	Ggr 1		Ggr 2		Ggr 3	
	M	*SD*	*M*	*SD*	*M*	*SD*
MD children	.91	1.15	2.41	1.65	4.17	1.59
MN children	12.27	5.99	19.41	2.41	25.61	6.43

Note. Number of problems = 28. Ggr1= Grade group 1 (Grades 2-3); Ggr2 = Grade group 2 (Grades 4-5); Ggr3 = Grade group 3 (Grades 6-7); MD = Children with mathematical difficulties; MN = Children without mathematical difficulties.

achievement groups as between-subjects factors and STRAT score as the dependent variable. The test indicated achievement group significant effect, $F(1, 132) = 294.51$, $p < .01$ ($R^2 = .688$). Furthermore, ANOVAs were performed to determine whether the STRAT scores of MN and MD children differed in the three grade groups. The analysis indicated significant differences for all three groups: $F_{Gr2\text{-}3}$ (1, 42) = 76.27, $p < .01$ ($R^2 = .636$); $F_{Gr4\text{-}5}$ (1, 42) = 441.18, $p < .01$ ($R^2 = .911$); $F_{Gr6\text{-}7}$ (1, 44) = 240.48, $p < .01$ ($R^2 = .842$).

Correlation analysis (Pearson) was also performed at three levels: all participants, grade group, and achievement group. Results of the analysis indicated significant correlations between STRAT scores and PSI scores when all children in the study were included, $r(132) = .763$, $p < .01$. Furthermore, significant correlations emerged across all three grade groups: r_{Ggr1} (44) = .500, $p < .01$; r_{Ggr2} (42) = .823, $p < .01$; r_{Ggr3} (44) = .748, $p < .01$. At the achievement group level, the analysis indicated significant correlations for MN children, r (67) = .516, $p < .01$, and significant correlations for MD children $r(67) = .322$, $p <.01$.

More specifically, unlike MN children, the MD children rarely used private speech-decomposition strategy combinations. The silence-decomposition strategy combination was mainly used by MN children. The most striking difference between MN children and MD children, however, is related to the silence private speech-retrieval strategy combination. Thus, we observed that MN children used this combination far more often than the MD children and that the difference between the two achievement groups increased with grade.

Consistent with earlier research, the results indicated that that mathematical development is reflected in use of a rich and varied repertoire of task-specific strategies, which become more efficient over time as a result of internalisation (Carpenter & Moser, 1982; Geary, 1993; Ostad, 1998). In comparison, MD children were less likely to use the more internalised, less audible forms of private speech, suggesting a possible link between strategy development and private speech internalization. In accordance with research in the field of private speech, the results suggest that successful mathematical development might be a function of efficiency in the production of task-relevant private speech (Berk & Landau, 1993; Harris, 1986; Hecht et al., 2001; Winsler & Naglieri, 2003). It could be argued that since MD children make less use of internalised private speech than their mathematical-typical counterparts, their MD can at least partly be attributed to a failure to use self-directed language to guide strategy use. (For further details from the study, see Ostad and Sorensen, 2007.)

Relationship Between Private Speech and Phonological Awareness

In general terms, phonological awareness relates to the auditory and oral manipulation of sounds, and refers to the ability to analyze the sound structure of oral language (Krajewski & Schneider, 2009). Phonological awareness can also be described as one's awareness of, and access to, the sound structure of oral language (Dolcos & Albarracin, 2014; Wagner & Torgesen, 1987).

No area of reading research has gained as much attention over the past three decades as phonological awareness. However, although it is well known that phonological awareness affects literacy development, its relation to mathematical competences is not well investigated (Krajewski & Schneider, 2009). Surprisingly, no published study on cognitive correlates of primary school mathematics performance has investigated the role of phonological awareness in private speech internalization.

Assessment of Phonological Awareness

Earlier research in this area did not use assessment instruments developed specifically for the mathematics domain; however, the relatively well-developed research literature in literacy (e.g., Chard & Dickson, 1999; Hecht et al., 2001) has provided insight into potential methods of assessing phonological awareness within the domain of mathematics.

A wide variety of tasks have been used to measure phonological awareness, including rhyming tasks, phoneme counting tasks, sound comparison tasks, blending tasks, segmentation tasks, and deletion tasks. There seems to be ample evidence that the tasks that require manipulation of phonemes are the most difficult. However, just how these tasks relate to each other is far from clear, and this makes it difficult to compare scores from different phonological awareness tasks (Adams & Hitch, 1997; Antony & Francis, 2005; Chard & Dickson, 1999; Hambleton, Swaminathan, & Roger, 1991; Stahl & Murry, 1994).

The test developed for this study was based on instruments used in the literacy domain (e.g., Anthony & Francis, 2002; Hambleton et al., 1991; Stahl & Murry, 1991). The test consisted of tasks related to two-, three-, and four-digit number words arranged in order of difficulty. Four tasks were associated with each number: blending, analysis, deletion, and substitution, in order of difficulty. All tasks require the subject to hold segmented phonological units in short-term memory and to access those representations consciously. In other words, the phonological awareness tasks required subjects to have a special type of access to phonological representations, namely *conscious access*, which may place special demands on retrieval mechanisms.

The total number of points were worked out in order of item difficulty and scored in the proportion 1: 2: 3 for number words with 2, 3, and 4 phonemes (digits), respectively. The participants gained points from the actual number word only if they answered all four tasks satisfactorily. The testing was ended after two subsequent mistakes. The total number of points (max 32 points) was transformed to scaled scores, named PHON scores. (For further details of the assessment process, see Ostad, 2013.)

Table 3 gives descriptive information (means and standard deviations) about the participants' phonological awareness expressed as PHON scores. The data are organized by grade group and by achievement group.

To examine whether the phonological awareness development of MD children showed different developmental patterns from those of MN children, we performed a univariate ANOVA, with achievement groups as between-subjects factors and PHON score as the dependent variable. The test indicated a significant achievement group effect, $F(1, 132) = 251.34, p < .01$ ($R^2 = .656$).

Tests of between-subject effects (ANOVAs) were performed to determine whether phonological awareness of MD and MN children differed in all three age groups. The analysis indicated significant phonological awareness differences in all three grade groups: F_{Ggr1} (1, 42) = 98.44, $p < .01$ ($R^2 = .701$); F_{Ggr2} (1, 42) = 207.66, p < .01 ($R^2 = .832$); F_{Ggr3} (1, 44) = 232.30, < .01 ($R^2 = 0.842$).

Correlation analysis was performed at three levels: all participants, grade group, and achievement group. The analysis indicated significant correlations between PSI scores and PHON scores when all children in the study were included, $r(132) = .767$, $p < .01$. Furthermore, the results indicated significant correlations across all three grade groups: r_{Ggr1} (42) = .551, $p < .01$; r_{Ggr2} (42) = .838, $p < .01$; r_{Ggr3} (44) = .720, $p < .01$. However, at the achievement group level, the analysis indicated significant correlations for MN children, r (65) = .585, $p < .01$, and non-significant correlations for MD children, $r(65) = .124$, $p > .05$.

Table 3

Means and Standard Deviations for Phnological Awarness Scores (PHON Score) by Grade and Mathematical Achievement Group

Mathematical Achievement Group	Ggr 1		Ggr 2		Ggr 3	
	M	*SD*	*M*	*SD*	*M*	*SD*
MD children	5.27	1.38	5.59	1.29	6.09	1.91
MN children	9.50	1.43	13.55	2.24	16.39	2.60

Note. Ggr1= Grade group 1 (Grades 2-3); Ggr2 = Grade group 2 (Grades 4-5); Ggr3 = Grade group 3 (Grades 6-7); MD = Children with mathematical difficulties; MN = Children without mathematical difficulties; DFS = Forward digital recall score; DBS = Backward digital recall score.

Among the MN children, there was a grade-depended shift from the lower to the higher levels of phonological awareness, whereas among MD children phonological awareness did not seem to progress beyond the level associated with the lower grades in MN children. These results are consistent with phonological deficit theory (Ramus & Szenkovits, 2008; Rasmussen & Bisanz, 2005; Wagner & Torgesen, 1987) in showing that young children with low or impaired phonological awareness seem to be at risk of developing problems with mathematics.

One of the most important insights from this study concerns developmental changes in the relationship between private speech and phonological awareness in children with and without MD. Thus, the MN children demonstrated a grade-dependent shift from use of audible private speech to use of silent private speech and from lower to higher levels of phonological awareness. In contrast, the development of MD was characterized not only by persistence of low phonological awareness but also by low levels of private speech internalization.

The most striking difference between children with and without MD, however, was related to the silence private speech-high phonological awareness combinations. Thus, this combination was used far more frequently by MN children than by MD children, and the difference between the two achievement groups became more marked as they progressed through primary school. More general analysis of the results revealed significant correlations between the children's developmental levels with respect to private speech and phonological awareness. (For further details from the study, see Ostad , 2013.)

Relationship Between Private Speech and Phonological Memory

The main line of research on phonological abilities focuses on phonological memory. Phonological memory is a form of memory that involves coding and storage of auditory information in short-term or working memory (Bishop, 2003; Swanson, Ashbaker, & Lee, 1996).

According to Wagner and Torgesen (1987), phonological memory deficits are one of the three main classes of phonological deficits. Arguably, phonological memory is important for solving arithmetic problems (Bull & Scerif, 2001; Fürst & Hitch, 2000). Consistent with this hypothesis, numerous studies have reported correlations between indicators of phonological skills and competence in mathematical computation (Adams & Hitch, 1997; Fuchs et al., 2006; Siegel & Ryan, 1989; Swanson & Sachse-Lee, 2001). In particular, phonological memory skills have been associated with the ability to do exact addition (Lemaire, Abdi, & Fayol, 1996), subtraction (Seyler, Kirk, & Ashcraft, 2003), and multiplication (Seitz & Schumann-Hengsteler, 2000). Furthermore, relationships have been reported between phonological memory and the ability to perform calculation procedures such as counting (Logie & Baddeley, 1987), retaining problem information (Fürst & Hitch, 2000) and holding interim results during counting (Berg, 2008). (For a review, see Raghubar, Barnes, and Hecht, 2010.)

In addition to correlational evidence from randomly selected samples, comparisons of children of differing mathematical abilities have shown that relative to children with grade-appropriate mathematical skills, children with MD tend to have poorer phonological memory whereas mathematically gifted children tend to have superior phonological memory (e.g., De Chambrier & Zeiger, 2018; Geary et al., 2000; Jordan et al., 2003; Mabbott & Bisanz, 2008; Ostad, 2013; Passolunghi & Cornoldi, 2008; Siegel & Ryan, 1989; Swanson & Jerman, 2006; Swanson & Sachse-Lee, 2001). These studies typically demonstrate that children with MD have lower digit span than children without MD. It is therefore surprising that no published study on cognitive correlates of primary school mathematics performance has investigated the contribution of phonological memory to private speech internalization. With this in mind, the main aim of our study (Ostad, 2015) was to determine how two indicators of phonological memory (forward digit span and backward digit span) are related to internalization of private speech and how this relationship varies with age and mathematical achievement levels.

Development of phonological memory and private speech was explored in two separate laboratory investigations*; age- and mathematical ability-related differences in phonological memory, and private speech* and *the relationship between phonological memory and private speech internalization.*

Assessment of Phonological Memory

Phonological memory is typically examined by serial recall tasks in which an individual is presented with a sequence of verbal items, such as spoken words or digits, and then asked to repeat them in either the same order (digit forward) or the reverse order (digit backward); for more details, see Swanson and Jerman (2006).

Several studies have hypothesised that factors of phonological memory may be loaded along a continuum (analogue and theoretical axis) from a passive storage system at one extreme, to an active storage system at the other (Cornoldi & Vecchi, 2000; Engle, Cantor, & Carullo, 1992; Passolunghi & Cornoldi, 2008; Swanson, 1994). In accordance with this hypothesis, digit forward tasks rely on a passive storage system and involve the recall of information without manipulating it in any way, which is closer to the passive pole. Conversely, digit backward tasks require more active processes and are those in which information is temporarily held while being manipulated or transformed, which is closer to the active pole (for more details, see Passolunghi and Siegel, 2004). The assessment of phonological memory skills in the present study was theoretically anchored to these hypotheses.

To standardise the measurements of digit span, we used two subtests of Math-Diagnostics (Ostad, 1987). The items in these subtests, a repetition of a dictated series of digits forward and digits backward, are comparable to the digit span from WISC-R (Undheim, 1977). Each series begins with two digits and keeps increasing in length, with two tasks of equal length.

Forward digit recall. The task requires the child to repeat a list of single-digit numbers in the same order as dictated the by the researcher.

Backward digit recall. The experimenter states a list of single-digit numbers and the child repeats them in reverse order. The procedure is similar to that of forward digit recall.

Table 4 presents descriptive information (means and standard deviations) about the performance of phonological memory measured as digit span forward and backward test results. The data are organised by grade group and by achievement group.

Tests of between-subject effects (ANOVAs) were performed to determine if the phonological memory test results of MD and MN children differed in all three grade groups. The analysis indicated significant differences in digit forward span test results across all three grade groups: $F_{Gr2\text{-}3}$ (1, 42) = 74.606, $p < .01$ ($R^2 = .640$); $F_{Gr4\text{-}5}$ (1, 42) = 52.250, $p < .01$ ($R^2 = .554$); $F_{Gr6\text{-}7}$(1, 44) = 40.631, $p < .01$ ($R^2 = .468$). The corresponding analysis indicated significant digit backward span differences: $F_{Gr2\text{-}3}$ (1, 42) = 63.117, $p < .01$ ($R^2 = .600$); $F_{Gr4\text{-}5}$ (1, 42) = 81.301, $p < .01$ ($R^2 = .659$); $F_{Gr6\text{-}7}$ (1, 44) = 79.148, $p < .01$ ($R^2 = .643$).

We performed two univariate ANOVAs (separate analyses for MD and MN children) with PSI scores as the dependent variable to determine whether PSI scores related differently to phonological memory test results between the two groups.

For the MD children, the analysis indicated a significant effect for grade group, F_{MD} (2, 67) = 10.494, $p < .01$, a significant effect for DF (forward digital recall) test results, F_{MD} (1, 67) = 28.501, $p < .01$, and a significant effect for DB (backward digital recall) test results, F_{MD} (1, 67) = 7.119, $p <.01$. The corresponding analysis of the MN children indicated significant effect for grade group, F_{MN} (2, 67) = 29.977, $p < .01$, a non-significant effect for DF, F_{MN} (1, 67) = .010, $p > .05$, and non-significant effect for DB, F_{MN} (1, 67) = 0.737, $p > .05$.

Correlation analysis (Pearson) was performed at three levels: all participants, grade group, and achievement group. The results indicated significant correlations between PSI scores and DF test results when all children in the study were included, r(132) = .645, $p < .01$. Furthermore, the analysis indicated significant correlations across all three grade groups: r_{Ggr1} (42) = .595, $p < .01$; r_{Ggr2} (42) = .664, $p < .01$; r_{Ggr3} (44) = .597, $p < .01$. At the achievement group level, the analysis indicated significant correlations for MN children, r (65) = .405, $p < .01$, and significant correlations for MD children, r(65) = .362, $p < .662$.

Table 4

Means and Standard Deviations for Phonological Memory Skills by Grade and Mathematical Achievement Group

Phonological Memory Category	Ggr 1				Ggr 2				Ggr 3			
	MD		MN		MD		MN		MD		MN	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
DFS	4.41	1.09	7.23	1.07	5.77	1.07	8.00	0.98	5.78	1.57	8.22	0.95
DBS	2.18	0.85	4.77	1.27	2.68	0.84	5.59	1.26	2.78	0.90	5.87	1.49

Note. Ggr1= Grade group 1 (Grades 2-3); Ggr2 = Grade group 2 (Grades 4-5); Ggr3 = Grade group 3 (Grades 6-7); MD = Children with mathematical difficulties; MN = Children without mathematical difficulties; DFS = Forward digital recall score; DBS = Backward digital recall score.

The corresponding analysis for the correlations between PSI scores and DB test results were as follows: When all children in the study were included, $r(132) = .638$, $p < .01$. Furthermore, the analysis indicated significant correlations across all three grade groups: r_{Ggr1} (42) = .511, $p < .01$; r_{Ggr2} (42) = .764, $p < .01$; r_{Ggr3} (44) = .639, $p < .01$. However, at the achievement group level, the analysis indicated significant correlations for MN children, r (65) = .529, $p < .01$, and non-significant correlations for MD children $r(65) = .678$, $p > .05$.

There were significant differences between children with and without MD on both the forward and backward digit span test in all the grade groups included in the study. These findings are consistent with the conclusion of earlier studies; namely, that phonological memory predicts the development of mathematics skills and understanding and that phonological memory is a domain-general precursor of mathematical achievement (Anderson-Day & Fernyhoug, 2015; Geary et al., 2000; Jordan et al., 2003; Mabbott & Bisanz, 2008; Passolunghi & Siegel, 2004; Swanson & Jerman, 2006; Swanson & Sachse-Lee, 2001).

One of the most important insights from this study concerns the relationship between phonological memory and private speech internalization. We wanted to determine whether the skills profiles of children with and without MD suggested that the relationships between phonological memory and private speech internalisation are a function of mathematical ability.

Comparisons of children at various developmental stages revealed differences between the two achievement groups. In contrast to the MD children, children without MD showed an age-dependent increase in the strength of the positive association between private speech internalization and phonological memory. We also observed relations between private speech and performance on digit forward and digit backward in children with and without MD. Regression analysis indicated that the contribution of forward digit span to variance in private speech internalization varied with mathematical achievement. Whereas digit backward span made a similar independent contribution to variance in private speech internalization in children with and without MD, the independent contribution of digit forward span was markedly higher in the group of MD children. Thus, the study provides evidence that the relationship between phonological memory and private speech internalization lies closer to a passive storages system (Cornoldi & Vecchi, 2000; Engle, 2002; Passolunghi & Siegel, 2004) in MD children than in MN children. (For further details from the study, see Ostad, 2015.)

Discussion

As a whole, the results from this project not only confirm that private speech internalization is related to mathematical achievement; they also highlight possible parallels between the contributions of strategies, phonological awareness, and phonological memory to subsequent mathematical achievement. However, there is no simple explanation for the patterns of the results reported here. It is possible that there are reciprocal relationships among mathematical performance, private speech internalization, phonological awareness, and phonological memory. If this is the case, a plausible explanation for the relationships observed in these studies is that they are all underpinned by variation in phonological skills, which are a function of the quality of an individual's phonological representations (Geary et al., 2012; Mabbott & Bisanz, 2008; Matuga, 2003; Ostad, 2015).

Overall, the results seem to provide evidence against the hypothesis that private speech internalization is a general cognitive process that is causally related to mathematical achievement. In contrast, they are consistent with the hypothesis that mathematical achievement is causally related to phonological abilities – which underpin private speech internalization – rather than being directly related to private speech internalization. Based on our findings, we believe that more private speech activities do not necessarily mean better problem solving. For private speech activities to have a positive impact on problem solving, they need to be anchored in developmentally appropriate cognitive skills. This hypothesis remains to be directly tested.

In terms of implications for educational practice, classroom teachers should be encouraged to allow their students to utilize private speech in their mathematics learning processes. Furthermore, consistent with suggestions developed from earlier private speech investigations (Anderson-Day & Fernyhough, 2015; Chard & Dickson, 1999; Matuga, 2003; Ostad &Askeland, 2008; Winsler et al., 2000), the various roles private speech play in cognition should be a compulsory component in teaching training programmes.

Limitations of the reported studies suggest that the findings should be accepted with caution. First, in interpreting the results, it is important to bear in mind that the measurements involved in the studies were concurrent in nature, and the causal direction of the relationships analyzed is not known. Second, the studies did not include data to determine the complexity of cognitive processes hidden behind the private speech category silence (e.g. Butterworth, 2005; De Chambrier & Zeiger, 2018; Geary et al., 2012; Winsler et al., 2006). Third, the presented studies used simple number fact problems in addition as their point of departure in the assessment of private speech internalization. Future investigations should address these gaps in the private speech literature, and include a broader selection of mathematical problems, word problems and number fact problems in subtraction and multiplication problems.

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