Using Virtual Manipulatives With Technology-Based Graphic Organizers to Support Students in Solving Proportion Word Problems

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Abstract

Proportional reasoning and knowledge of fractions are critical skills for completing algebra successfully, yet many students with mathematics learning disabilities (MLD) enter algebra classes without adequate prior knowledge of these skills. This study used a single-subject/case research, combined multiple-baseline and alternating-treatment design to determine the functional relation between the use of varied technology-based interventions (i.e., a virtual manipulative [VM], a technology-based graphic organizer [TBGO], and a combination of both [VM+TBGO]) and student accuracy in solving proportion word problems. Additional dependent variables included student independence and duration. Three high school students with MLD solved proportion word problems without assistance in the baseline phase and then randomly alternated between the three treatments during intervention. Overall, student accuracy and independence increased, and the time required to solve word problems decreased. All students preferred using technology to solve word problems and the VM+TBGO. Limitations, implications for practice, and suggestions for future research are discussed.

Keywords: Learning disability, mathematics intervention, technology-based intervention

Approximately 5 to 10% of students in the United States have a mathematics learning disability (MLD; Geary et al., 2012). Broad categories of mathematics that impede the learning of students with MLD include number sense, computational skills, and understanding math problems, symbols, and operations. These students may take an extended time to problem solve and rely heavily on teacher assistance to identify the appropriate steps towards finding a solution (Garnett, 1998). Subsequently, math concepts, math facts, and math procedures can be challenging for them students to learn (Doabler et al., 2012). Students may lack the prerequisite skills to understand a new math concept or more complex coursework, including pre-algebra or algebra (Impecoven-Lind & Foegen, 2010). Further, attentional difficulties, memory deficits, and language processing differences can negatively impact their learning of essential skills in K-12 mathematics (Shin & Bryant, 2015).

Teaching Proportions and Word Problems to Students With MLD

There are critical prerequisite skills learners must acquire for successful algebra completion. Two such skills include proportions (i.e., fractions) and word problems, which are often linked in the field. For instance, the mathematics curriculum for pre-algebra and algebra courses in K-12 education often combines proportions and word problems to assess

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student mastery of both the concept of proportions and its application (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

Knowledge of fractions and division in early grades is connected to later high school algebra achievement (Siegler et al., 2012). As a result, the National Mathematics Advisory Panel (2008) recommends that a mathematics curriculum spends a sufficient amount of time focusing on fractions and proportional reasoning as these areas are found to "have the broadest and largest impact on problem-solving performance" (p. xix). Although understanding fractions and proportional reasoning is an essential prerequisite for success in future algebra courses (Welder, 2006), many students with MLD enter pre-algebra and algebra courses without these skills.

Much like fractions, learning to solve word problems is a critical skill that students must develop during the formative years of their schooling. Such problems require multiple competencies on the part of a learner, including reading, visualizing, and analyzing information (Powell & Fuchs, 2018). However, word problems are challenging to navigate for students with MLD due to their difficulties interpreting the word problem, understanding the schema, and translating the problem into a mathematical equation (Shin & Bryant, 2017). Due to the significance of learning both fractions and word problems and the well-documented struggles students with MLD exhibit with each, multiple evidence-based practices have been established in the field to support these learners in each area, including manipulatives for teaching fractions (Butler et al., 2003) and schema-based instruction (SBI) for word problem instruction (Jitendra et al., 2002).

Manipulatives for Students With MLD

Fortunately, a growing body of research has identified practices and strategies for providing high-quality mathematics instruction to students with MLD. These evidence-based practices include explicit and direct instruction (Bender, 2009; Swanson & Hoskyn, 2001), schema-based instruction (e.g., Jitendra et al., 2002), visual and schematic representations (Jitendra et al., 2015), graphic organizers (Ives, 2007), and manipulatives (e.g., Witzel, 2005). Manipulatives, which come in two forms – concrete and virtual – allow students to be actively engaged in the learning process (Bouck & Park, 2018; Satsangi & Bouck, 2014). Most studies support manipulatives through the Concrete-Representational-Abstract (CRA) framework in which concrete manipulatives (e.g., geoboards, pattern blocks) serve as a visual representation of an abstract idea and can scaffold a student's understanding of this concept. The student can then move to two-dimensional representations before solving abstract problems without the visual representations. In algebra, secondary-level students must quickly move into the abstract phase of the CRA sequence, as symbolic notation is an essential understanding for success in such courses (Strickland & Maccini, 2012).

Mathematics educators of students with and without disabilities have increasingly used concrete (CMs) and virtual manipulatives (VMs) as visual representations across K-12 (Bouck & Park, 2018; Moyer et al., 2002), and research suggests that the latter may be more age-appropriate for secondary learners (Satsangi & Miller, 2017). VMs are defined as an "interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer et al., 2002, p. 373). As education moves toward an increasingly technology-based environment, the use of VMs is becoming more apparent in classrooms.

With greater use, researchers sought to study VMs within the CRA sequence to understand the best way to transition students to abstract learning. Some studies assessed students who transition directly from VMs to abstract notation (i.e., a VA teaching sequence; Bouck et al., 2020; Satsangi & Bouck, 2014; Satsangi et al., 2018), while others emulated the CRA sequence using a Virtual-Representational-Abstract (VRA) model (e.g., Bouck et al., 2017). Although studies with both approaches have returned promising findings thus far, additional research is needed on VMs to support students with MLD across all subjects, including word problems and proportions. To further advance research on their benefits on these topics, VMs are often studied in conjunction with other proven strategies, such as schema-based instruction.

Schema-Based Instruction for Supporting Learners With MLD

Students often need support in understanding the structure of a mathematics problem and organizing the information presented within it (van Garderen et al., 2013). Schema-based instruction (SBI) is an evidence-based practice for students with MLD that primes the underlying math problem's structure via a diagram (Jitendra et al., 2015; Powell & Fuchs, 2018). For example, multiplicative schemas allow students to find the quantitative relationship between the numbers when teaching proportional problem-solving in upper-elementary and middle school grades. SBI can be adapted to include the use of a paper-based graphic organizer to support organization. Just as concrete manipulatives have their virtual counterpart, SBI can be presented in a virtual medium and utilize a technology-based graphic organizer (TBGO) with or without VMs to represent the information and the unknown. In word problem instruction, a paper-based graphic organizer or TBGO can provide a visual way to organize the information (Powell & Fuchs, 2018).

Multiple studies have examined instruction featuring VMs with SBI to teach students with disabilities. For example, Root et al. (2017) investigated the effects of modified SBI with CMs, VMs, and schematic diagrams (either paper or virtual) on mathematical word problem-solving skills for three students with autism spectrum disorders (ASD) and moderate intellectual disabilities. All students' performance improved following modified SBI, and all three students enjoyed both concrete and virtual or technology-based supports. While this study was conducted with students with ASD, the results are consistent with other studies comparing CMs and VMs among students with learning disabilities (e.g., Satsangi et al., 2016), suggesting that technology can promote student independence in mathematical problem-solving, regardless of diagnosis. Further, Saunders (2014) used SBI virtually to provide students with ASD and intellectual disabilities support in word problem instruction. VMs and CMs were used to reinforce procedural knowledge and organize information in a VMGO. Elsewhere, Reneau (2012) used VMs combined with SBI with the CRA sequence to teach fraction word problems to students with MLD. Despite these studies, the authors found no published research that assesses the value of emerging forms of VMs and SBI in teaching students with MLD the critical skills of proportion word problems.

The Current Study

This study sought to assess the value of virtual manipulatives (VMs) and a technology-based graphic organizer (TBGO) to teach proportion word problems using explicit instruction to secondary learners with MLD. We assessed the value-added of VMs and TBGO interventions separately to determine whether these interventions were compatible together as part of a larger intervention package. Specifically, we sought to answer the following research questions:

(a) To what extent does a VM, TBGO, or a VM+TBGO combined with explicit instruction improve and maintain accuracy for secondary students with MLD when solving proportion word problems?
(b) Will a VM, TBGO, or a VM+TBGO differentially affect students' independence in solving proportion word problems?
(c) Do secondary students with MLD solve proportion word problems faster when using the VM+TBGO than when they use VM or TBGO only?

Method

A combined single-subject/case research design (SSRD; Ledford & Gast, 2018) was used. A multiple-baseline design examined the functional relation between students' accuracy when solving proportion word problems when provided with varying technology-based supports. A staggered introduction of the intervention provided experimental control and internal validity. The order in which the participants entered the intervention was determined randomly, controlling for Type 1 error. In addition, an alternating-treatment design compared student performance when using three technology-based interventions: VM, a TBGO, and VM+TBGO. Rapid and random alternations between conditions controlled for order and carry-over effects while exploring differential effects of three comparable technology-based interventions. The study meets the four standards for SSD research (Kratochwill et al., 2010).

Participants

Prior to the onset of the study, all approvals from both school district and university human subject review boards were obtained. A criterion sampling was used to select participants from a high school in the Mid-Atlantic region in the United States. Selection criteria included a student with an MLD, enrollment in Algebra I, math instruction in a self-contained setting, and a score of 50% or below on the pre-assessment, which contained proportion word problems. Students with MLD were those classified by their school district's eligibility process, which used a response-to-intervention (RTI) approach to identify students (Fuchs & Fuchs, 2006). The lead researcher distributed consent forms to 20 students who met the criteria. Of the five students who returned the consent forms, three, Gregory, Kristian, and Mark, scored below the 50% pre-assessment threshold, thus meeting all requirements for participation (see Table 1).

Table 1
Student Participant Characteristics

Student	Age	Disability	Ethnicity or Race	ELL Level	FSIQ Range	WJIV Broad Math Range	SST Scores (8th Grade/Algebra I)
Gregory	15	SLD/OHI	African-American or Black	N/A	Low Average[a]	Very Low	337/408
Kristian	14	SLD	Hispanic or Latino	4	Average[b]	Low	269/N/A
Mark	17	SLD	African-American or Black	3	Below Average[b]	Very Low	327/404

Note. ELL = English language learner, as measured by the World Class Instructional Design and Assessment (WIDA, 2015). OHI = other health impairment. SLD = specific learning disability. SST = State standardized test. WJIV = *Woodcock Johnson Tests of Achievement, Fourth Edition* (Schrank et al., 2014). A score of 400 or higher is considered passing for the SST.

[a]Full Scale Intellectual Quotient (FSIQ) measured by the *Cognitive Assessment System, First Edition* (CAS; Naglieri & Das, 1997). [b]FSIQ measured by *Kaufman Assessment Battery for Children, Second Edition* (KABC-II; Kaufman & Kaufman, 2004).

Gregory

Gregory was a 10-grade student with MLD and a secondary disability of other health impairment (OHI). His disability impacted reading, mathematics, and attention. He had a mathematics calculation goal in his Individualized Education Program (IEP). Gregory's Full-Scale Intelligence Quotient (FSIQ) range, math performance levels, and scores on the State Standardized Test (SST) in mathematics are found in Table 1.

Kristian

Kristian was a ninth-grade student with MLD with reported challenges in reading, writing, mathematics, and attention. Kristian had a mathematics calculation goal on his IEP. Kristian, whose first language is Spanish, scored at the expanding English proficiency level. Kristian's FSIQ range, math performance levels, and scores on the SST in mathematics can be found in Table 1.

Mark

Mark was a 10-grade student with MLD with reported reading, writing, and mathematics challenges. He had a mathematics reasoning goal in his IEP. Mark, whose first language is Amharic, scored on the developing level for English proficiency. Mark's FSIQ range, math performance levels, and scores on the SST in mathematics can be found in Table 1.

Setting and Interventionist

At the time of the study, the participating school's population of 1,900 students reported the following demographics: 44% Hispanic, 22% African American, 20% Caucasian, 9% Asian, 4% two or more races, and 1% American Indian/Alaskan Native or Hawaiian Native/Pacific Islander. In 2017, 49.17% of students at the school received free or reduced-price lunch. The school is located in an urban setting in the Mid-Atlantic.

The female Caucasian interventionist (first author) was a former certified high school math and special education teacher and graduate research assistant pursuing a Ph.D. in special education. She worked with each student participant in a one-on-one setting during students' mathematics classroom time in a small, isolated office located in their school. This office was approximately eight feet by 15 feet and contained a round table with four chairs.

Independent Variables

Three independent variables (IV) were used in the study: VM only, TBGO only, and VM+TBGO (see Figure 1). All IVs, or interventions, were comparable as they asked students to answer the same types of proportion word problems, provided students with identical feedback, and had similar features for computing the answer.

The VM+TBGO is an app-based software tool by Math Playground (2018) and is available for free online and accessible by computer or tablet. For the current study, the interventionist used the *Solve It! Thinking Blocks* activity and selected the "ratio and proportion" practice problems. This study used two types of problems that compare two ratios only: "missing quantity" and "find the total." The following is an example of a missing quantity problem: "The

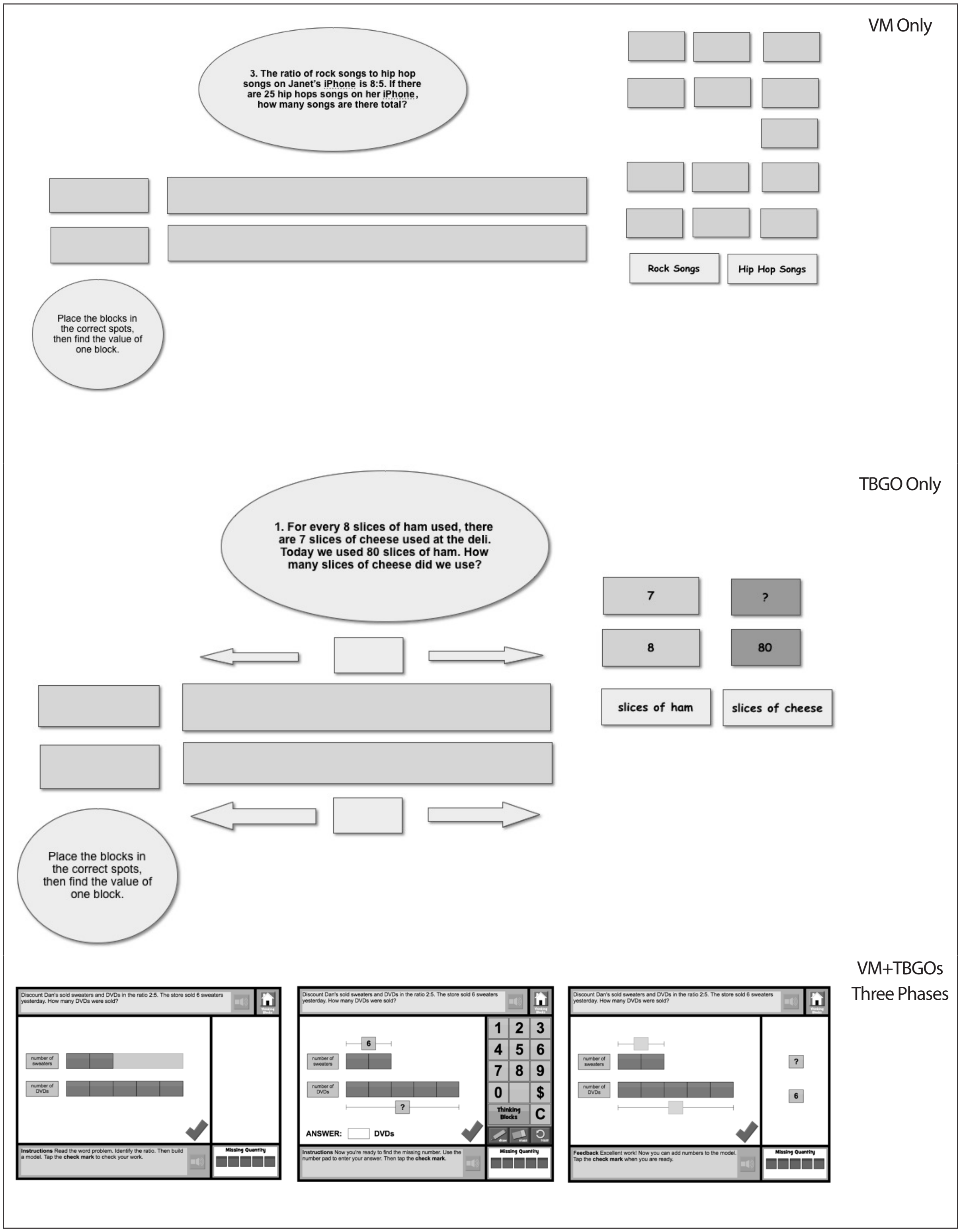

Figure 1

Virtual Manipulative Example Problem; Technology-Based Graphic Organizer Example Problem

Note. Phases of the VM+TBGO from Math Playground (2008): VM, TBGO, Computation (combined).

ratio of sprinters to long-distance runners is 5 to 4 on the track team. The track team has 35 sprinters; how many long-distance runners are there?" An example of a "find the total" problem is "For every 6 calendars Zoey sells, Nancy sells 5. The two friends sold a total of 99 calendars last week. How many calendars did Nancy sell?"

The VM+TBGO presented the word problem to the student in three different steps: (a) the VM step, which required the user to use VMs to set up the problem; (b) the TBGO step, which required the user to fill in the missing pieces of the TBGO; and (c) the computation step, which required the user to compute the problem.

The VM only and TBGO only were created by the interventionist using Inspiration© software. These two interventions followed a setup to their respective phases similar to the VM+TBGO. In Inspiration©, the student could click and drag the provided virtual blocks to the correct label.

Instructional Lesson

The interventionist taught an explicit instructional lesson across two 25-minute sessions prior to the start of any of the interventions mentioned above. The lesson focused on how to solve word problems involving proportions, a Grade 6 and 7 standard, and a prerequisite skill needed for standards in Grade 8 and up. Therefore, this lesson was a review for the participating students. The lesson used direct instruction (Bender, 2009) to teach the following elements: (a) vocabulary instruction on ratio and proportions, (b) identification of important elements provided by word problems, (c) how to use an iPad and stylus, (d) how to use the VM only to solve word problems, (e) how to use the TBGO only to solve word problems, and (f) how to use the VM+TBGO to solve word problems.

After the two sessions, students were given a six-question quiz with three problems of each type: find the missing quantity and find the total. Students needed to score 50% of problems correctly before advancing to the intervention phase. All students scored at least a 50% on the quiz (scores of 50%, 85%, and 50% were reported).

Materials

The students had access to an iPad and Apple© stylus pencil to access the interventions (VM only, TBGO only, VM+TBGO). Students could use their fingers or stylus. The VM+TBGO provided accessible sound effects.

Word Problem Probes

All word problem probes, except for VM+TBGO, were researcher-created and followed the Common Core State Standards (CCSS; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) for ratios and proportions found in sixth- and seventh-grade mathematics. The authors evaluated all word problem probes for clarity, consistency, and similarity of format. They were also shared with a special educator in the mathematics department of the students' school to ensure that they were designed appropriately and aligned with the age group and needs of the student participants.

Each word problem probe, in all phases, consisted of four problems, two of each of the following types: find the missing quantity and find the total. No word problems were repeated within and across probes. The probes were printed on paper in baseline and generalization; in all other phases, they were presented on the iPad. Students were given paper to show their work.

The Math Playground's *Solve It! Thinking Blocks* in the VM+TBGO generated the word problem probes. For each of those probes, the lead author selected from the two corresponding categories: "missing quantity" and "total." In the VM only and TBGO only interventions, the lead author created the word problem probes; these probes were identical in type and format to the word problems created by MathPlayground's *Solve It! Thinking Blocks* in the VM+TBGO intervention. The VM+TBGO included a read-aloud option that students could use at any time. All questions and prompts in all interventions were read aloud by the interventionist, or the read-aloud option was used.

Dependent Variables

The dependent variables were measured using a data collection sheet and included the following: (a) percentage of accuracy in solving word problems, (b) percentage of prompting needed (called independence) when solving word problems, and (c) the time students needed to answer word problem probes. A data collection sheet was used to record student accuracy, the number of prompts required by the VM+TBGO or the interventionist (in the VM only and TBGO only conditions), and the time needed to complete the word problem probe, modeled after previous studies involving VMs (e.g., Satsangi et al., 2018).

Percentage of Accuracy

The percentage of accuracy was represented by the number of problems (out of four) a student answered correctly on each probe. During baseline and generalization, no prompts were provided. These problems were scored as correct or incorrect based on the final answer verified by the answer key. In all other phases, the problem was considered correct or incorrect by the final answer presented by the student. This answer was recorded on the digital workspace in the VM and TBGO phases and placed into the answer box in the VM+TBGO. An answer key was used to score answers in the researcher-created VM only and TBGO only conditions. The VM+TBGO app notified the student and interventionist of a correct or incorrect answer. A percentage of accuracy was calculated by dividing the number of correct answers by four.

Percentage of Independence

The percentage of independence for each probe was calculated based on the type of prompts that each student received for each word problem. In all interventions, the type of prompt was considered a P1, P2, or P3, the levels indicating the magnitude of the prompt needed by the student. The interventionist provided verbal feedback from a script on the data collection recording sheet created using the written VM+TBGO feedback prompts that appeared on the screen.

Any prompts that appeared on a screen were read to the student. A P1 prompt indicated that the interventionist or technology tool provided verbal (read by the interventionist) or written feedback (provided by the VM+TBGO and read aloud using the read-aloud option) to the student. A P2 prompt indicated that both verbal or written feedback was provided by the interventionist or VM+TBGO, with the interventionist pointing to the place on the screen where the student was making a mistake. Finally, a P3 prompt indicated that the interventionist modeled the problem either by controlling the program, writing out steps on paper, or providing multiple points using a finger on the screen (e.g., Satsangi et al., 2018). In the instance of a P3 prompt, the problem was automatically considered incorrect, and the student was asked to complete a problem correctly.

In the VM+TBGO intervention, embedded feedback was presented in each phase of the word problem from the app. If the feedback was corrective, the interventionist recorded the number of times the student attempted to correctly set up the problem using the appropriate P-level prompt. In the VM only and TBGO only interventions, feedback paralleled that provided in the VM+TBGO, when it was provided, and how it was provided (verbally). Specifically, the interventionist carefully observed the student and their work and delivered the scripted feedback printed on the data collection recording sheet. If feedback was given to correct the setup or calculation of a problem, it was considered a P1 prompt, as long as it was not supplemented with pointing to the screen.

The percentage of independence was calculated using the following formula: the number of prompts needed overall (regardless of prompt level) divided by 12 and multiplied by 100. There were four problems on each probe, and participants could demonstrate independence for each of the three steps (see Figure 1) required to solve each problem; therefore, a total of 12 was the denominator. If three prompts were needed in any step, the entire problem was considered incorrect.

Time Required

The duration of each probe was measured by recording the time when the student started reading the first word problem in the probe and recording the time when the student finished the probe. Students did not take any breaks during a probe.

Data Collection Procedures

Data collection began with the three student participants on the same day and continued over four months. Students met with the interventionist approximately every other school day. Students were selected for sessions using a random number generator. In all phases, accuracy, independence, and time was recorded. Additionally, the interventionist adhered to the fidelity checklist and procedures for documentation on the data collection sheet.

Baseline Phase

In the baseline phase, the student participants were given a minimum of five probes to assess level stability in data and to address the SSRD quality indicators as described by Horner et al. (2005). No prompts were given during this phase. After each baseline probe, the interventionist recorded the percentage of accuracy and time.

Intervention Phase

Once data of Participant 1 had shown stability in baseline with at least five data points, the interventionist provided the participant with the instructional lesson(s). Once Participant 1 showed an immediate effect with stability in their intervention data, Participant 2 was provided the instructional lesson, and likewise for Participant 3. Each participant alternated between three interventions following the instructional lesson: VM only, TBGO only, and VM+TBGO. Each intervention was assigned a number and randomly chosen at the start of the intervention phase. The randomly selected intervention probe was administered according to the randomized schedule, with no intervention repeated more than twice in a row to avoid the risk of cyclical variability (Kratochwill et al., 2010).

Best Treatment Phase

A best treatment intervention was selected based on the students' percentage of accuracy scores for each probe. A "superior treatment" phase may lessen the threat of multi-treatment interference, as it measures the intervention in isolation rather than when it is measured in alternation with other interventions (Ledford & Gast, 2018). If students had equal accuracy scores between the interventions, they were asked which of the interventions they preferred for consideration in the best treatment phase. The student then completed the best treatment phase for a minimum of five probes, or until stability was reached in this phase.

Maintenance Phase

Following the best treatment phase, the student participant entered into a 10-day minimum hiatus from the best treatment phase. After the hiatus, the student completed a minimum of three additional sessions of best treatment probes. This phase followed the same procedures as the best treatment phase to assess students' performance after a break.

Generalization Phase

After each sstudent completed the maintenance portion of the best treatment phase, they completed a generalization probe with the researcher to assess the students' abilities to solve proportion word problems without manipulatives or graphic organizers. The probe was identical in length and format to the pre-assessment given at the beginning of the study. Student participants were only able to use paper and a graphing calculator to answer the problems.

Inter-Observer Agreement and Treatment Fidelity

Inter-observer agreement (IOA) data for each dependent variable and treatment fidelity were collected in a minimum of 30% of all sessions per participant in all phases (Kratochwill et al., 2010). A trained colleague independently scored student probes using the procedures outlined in the training and checked off the fidelity checklist to ensure that the procedures were followed identically for each administered probe. The interventionist and colleague compared data collection sheets and noted the number of agreements and disagreements and the level of prompt noted. A percentage was calculated using the following formula: agreements divided by the total number of agreements plus disagreements, multiplied by 100. The percentage agreement for IOA across all DVs was 99.7%.

The treatment fidelity checklist consisted of procedural activities that the interventionist followed each time a participant began a probe. The trained colleague measured implementation fidelity using the fidelity checklist in 30% of all sessions in all phases. Treatment fidelity percentage was calculated by the number of steps met, divided by the total number of steps, multiplied by 100. The fidelity of treatment implementation for all three students was 100%.

Social Validity

The social validity of the interventions was measured via individual interviews with the participants. Presented as a semi-structured interview, questions were conducted verbally, and audio recorded. Each student interview lasted approximately 15 minutes. Questions focused on the students' preferences for the interventions and the features of the interventions that worked best for them.

Data Analysis

The data collected in this study underwent multiple forms of analysis: visual analysis, calculation of the percent of non-overlapping data (PND), and descriptive statistics. A visual analysis was conducted by examining the plotted data in a graph format. The lead researcher visually inspected the graphs for assessing basic effects in SSD: (a) level change, (b) trend, (c) variability, (d) immediacy of effect, (e) consistency, and (f) overlap of data points (Kratochwill et al., 2010).

To conduct a *level analysis* within a phase, the mean value of the data was calculated, and a mean line was drawn on the graph parallel to the x-axis. The *trend* was defined as the slope of the best-fitting straight line within each phase (Kratochwill et al., 2010). The *variability of the phase* was calculated by examining the number of data points that fell within a 25% range of the median. If 80% of the data points fell within that range, the data for that phase were considered "stable" (Ledford & Gast, 2018). The *immediacy of effect* between baseline and intervention phases was considered when there was an immediate change in level between the last three data points in one phase and the first three data points in the next phase. In addition, the consistency of data patterns in similar phases (e.g., baseline, intervention, maintenance, generalization) was determined (Kratochwill et al., 2010). Finally, *percent of non-overlapping data* (PND) was used to evaluate the overlap between each intervention and the baseline phase. It was calculated by finding the highest value in one phase and then counting the number of values that fall above that data point in the adjacent phase multiplied by 100 (Scruggs & Mastropieri, 1998). Overall, the higher the percentage, the likelier the intervention impacted the target behavior.

Descriptive statistics were used to evaluate the level of independence per intervention and per student. The more independent the student was in completing the probes (i.e., limited, if any, prompts), the higher the independence percentage.

Students' responses from interviews were analyzed by the lead researcher and a colleague who reviewed the audio recordings, transcription, and analysis to confirm the interpretation of the responses. Transcripts were coded according to Carspecken's (1996) model of analysis. Codes were created to identify interview data that was meaningful for analysis and informed the social validity of the study. These codes (e.g., preference for learning mathematics, preference for treatment, preference for features of treatment) were grouped and themes emerged from the data. The data were grouped based on student responses and were analyzed separately. These data were used to determine the social validity of the treatments used. The lead researcher read a summary of the analysis back to the students as a form of member checking (Birt et al., 2016). Students confirmed that the researcher correctly analyzed their responses.

Results

Overall, accuracy and independence increased or remained the same, and time decreased for the three students. Results per participant are provided below for each measure.

Accuracy

As shown in Figure 2, there was strong evidence of a functional relation across all the participants. The visual analysis of data demonstrated an immediate change in level between baseline and intervention phases resulting in 100% PND across the participants.

Gregory

Gregory's baseline level for accuracy (see Table 2) was 10% (*SD* = 13.69). The trend was flat with medium variability. Gregory showed an immediate change in accuracy as he entered the intervention phase. The overall level across all three interventions was 83.33% (*SD* = 15.43), with a flat trend and high variability. His average score with the TBGO only intervention was 75% (*SD* = 17.67), 85% (*SD* = 13.69) with the VM only intervention, and 90% (*SD* = 13.69) with the VM+TBGO. Gregory's performance was consistently higher in the intervention phase than in the baseline. PND was 100% for each of the three interventions indicating no overlap and high effectiveness of all interventions.

In the best treatment phase, Gregory's accuracy level using the VM+TBGO was 95% (*SD* = 11.18, see Table 2), with a flat trend and medium variability. After the two-week hiatus, Gregory completed the maintenance phase consistently, scoring 100% (*SD* = 0) with a flat trend and no variability. When all interventions were removed, Gregory's generalization score was 75%. Overall, his scores during best treatment, maintenance, and generalization phases were consistently higher than in baseline.

Kristian

Kristian's baseline level for accuracy was 3.57% (*SD* = 9.45), demonstrating a flat trend and medium variability. Kristian showed an immediate change inaccuracy with the overall level across all three interventions of 98.33% (*SD* = 6.45), flat trend, and low variability. His average score with the TBGO only intervention was 100% (*SD* = 0), 100% (*SD* = 0) with the VM only intervention, and 95% (*SD* = 11.18) with the VM+TBGO. Kristian's performance was consistently higher in the intervention phase than in baseline. PND was 100% for each of the three interventions indicating no overlap and high effectiveness of all interventions. Kristian chose the VM only intervention for the best treatment and maintenance phases because his accuracy was identical for both the TBGO only and VM only interventions.

Table 2
Average Percentage of Accuracy in Each Phase and Each Treatment

Student	Baseline	Intervention			Best Treatment[a]	Maintenance[a]	Generalization
		TBGO	VM	VM + TBGO			
Gregory	10%	75%	85%	90%	95%	100%	75%
Kristian	3.5%	100%	100%	95%	100%	100%	75%
Mark	0%	90%	95%	95%	100%	100%	100%

Note. [a]Best treatment and maintenance varied for each student: Gregory = VM+TBGO, Kristian = VM, Mark = VM+TBGO.

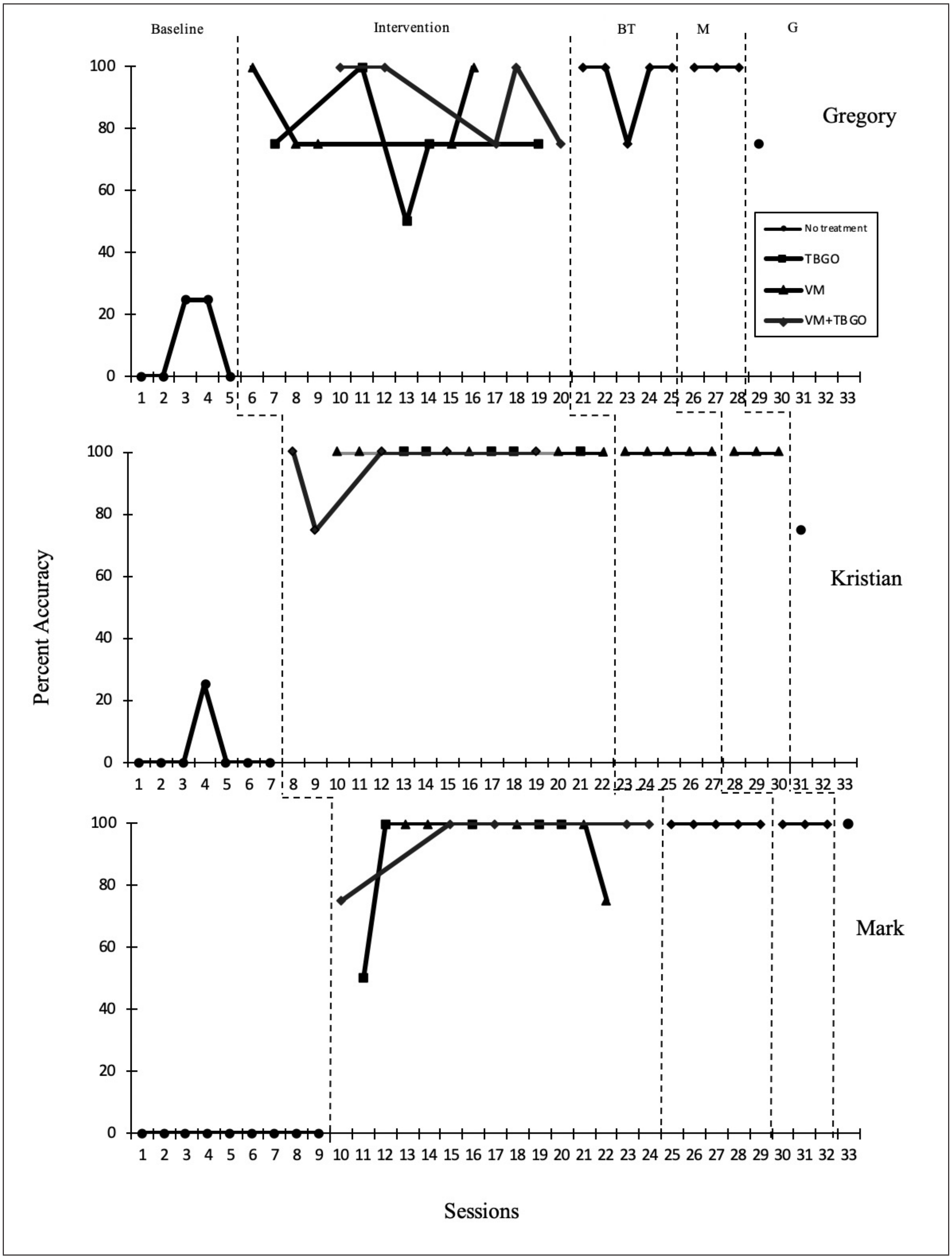

Figure 2

Percentage of Accuracy Per Probe for the Three Participants Across Five Phases: Baseline, Intervention, Best Treatment, Maintenance, and Generalization

Note. BT = Best Treatment, M = Maintenance, G = Generalization.

Kristian's level was 100% (*SD* = 0) in the best treatment phase with a flat trend and no variability. After the two-week hiatus, Kristian completed maintenance probes consistently, scoring 100% (*SD* = 0) with a flat trend and no variability. Kristian's generalization accuracy score was 75% when all interventions were removed. Overall, his scores during best treatment, maintenance, and generalization phases were consistently higher than in baseline.

Mark

Mark's baseline data for accuracy was stable with the 0% level and flat trend. He showed an immediate change inaccuracy with the overall mean across three interventions of 93.33% (*SD* = 14.84), different trends, and medium-to-high variability across interventions. His average score with the TBGO-only intervention was 90% (*SD* = 22.36), showing an upward trend and high variability. His average score with the VM-only intervention was 95% (*SD* = 11.18), with a downward trend and medium variability. Finally, his average score in the VM+TBGO was 95% (*SD* = 11.18), with an upward trend and medium variability. Mark's performance was consistently higher in the intervention phase than in baseline. PND was 100% for each of the three interventions indicating no overlap and high effectiveness of all interventions. Mark chose the VM+TBGO intervention for the best treatment and maintenance because his accuracy was similar in VM only and VM+TBGO interventions.

Mark's level was 100% (*SD* = 0) in the best treatment phase with a flat trend and no variability. After the two-week hiatus, Mark completed maintenance probes consistently, scoring 100% (*SD* = 0) with a flat trend and no variability. Mark's generalization accuracy score was 100% when all interventions were removed. Overall, his scores during best treatment, maintenance, and generalization phases were consistently higher than in baseline.

Independence

The students' need for assistance from the researcher for the VM only or TBGO only intervention or the VM+TBGO program decreased overall as students became more independent in their problem-solving over time. No assistance was given in the baseline or generalization phases. The students received prompts when needed during the intervention, best treatment, and maintenance phases. In those three phases, the level and number of prompts decreased. That is, independence increased or stayed consistent for all three participants (see Table 3).

Gregory

Overall, Gregory's independence across phases increased over time. During the intervention phase, Gregory's independence ranged from 67% to 92% across all three interventions. In the TBGO intervention, Gregory's average percentage of independence was 83.4% (*SD* = 10.21, see Table 3). In the VM-only intervention, Gregory's average percentage of independence was 78.2% (*SD* = 7.16). In the VM+TBGO intervention, Gregory's average percentage was 88.4% (*SD* = 4.93). The most frequent type of prompting for all three interventions was P1, which was a prompt from the app that asked the student to retry a particular step of the problem-solving process.

In the best treatment phase, Gregory's independence, using the VM+TBGO intervention, ranged from 75% to 100%, with Gregory's last probe at 100% independence. The average rate of independence during this phase was 86.6% (*SD* = 9.61); Gregory required nine P1 level prompts during this phase. During the maintenance phase, which used the VM+TBGO intervention, Gregory's independence increased to an average of 91.67% (*SD* = 8.50) with a range of 83% to 100%. He required two P1 level prompts and two P2 level prompts. Finally, he received no support or interventions in the generalization phase as the procedures dictated that students work completely independently during the phase.

Kristian

Overall, Kristian's independence across phases stayed consistent. During the intervention phase, Kris-

Table 3

Average Percentage of Independence When Solving Proportion Problems in Each Phase

Student	Intervention			Best Treatment[a]	Maintenance[a]
	TBGO	VM	VM + TBGO		
Gregory	83.4%	78.2%	88.4%	86.6%	91.7%
Kristian	93.4%	95.2%	81.8%	93.4%	94.7%
Mark	86.8%	93.4%	98.4%	100%	100%

Note. [a]Best treatment and maintenance intervention varied for each student: Gregory = VM+TBGO, Kristian = VM, Mark = VM+TBGO.

tian's independence ranged from 67% to 100% across all three interventions. In the TBGO intervention, Kristian's average percentage of independence was 93.4% (*SD* = 10.85, see Table 3). In the VM only intervention, Kristian's average percentage of independence was 95.2% (*SD* = 4.38). In the VM+TBGO intervention, Kristian's average percentage of independence was 81.8% (*SD* = 10.89). The most frequent type of prompting for all three interventions was P1, which was a prompt from the app that asked the student to retry a particular step of the problem-solving process.

In the best treatment phase, which used the VM-only intervention, Kristian's independence ranged from 83% to 100%, with the last probe at 100% independence. During this phase, the average rate of independence was 93.4% (*SD* = 7.06), and Kristian required four P1 level prompts and one P2 level prompt. During the maintenance phase, which used the VM only intervention, Kristian's independence increased to an average of 94.67% (*SD* = 4.62) with a range of 92% to 100%. The trend was consistent from best treatment to the maintenance phase. Finally, in the generalization phase, Kristian received no support or interventions as the procedures dictated that students work completely independently during the phase.

Mark

Overall, Mark's independence across phases increased over time. Mark's independence ranged from 50% to 100% across all three interventions in the intervention phase. In the TBGO intervention, his average percentage of independence was 86.8% (*SD* = 20.96, see Table 3). In the VM-only intervention, Mark's average percentage of independence was 93.4% (*SD* = 7.06). In the VM+TBGO intervention, Mark's average percentage of independence was 98.4% (*SD* =3.58). The most frequent type of prompting for all three interventions was P1, which was a prompt from the app that asked the student to retry a particular step of the problem-solving process.

In the best treatment phase, which used the VM+TBGO intervention, Mark was 100% independent in all five probes. During the maintenance phase, which used the VM+TBGO intervention, Mark was 100% independent in all probes. Finally, no support was given by the researcher or via intervention in the generalization phase.

Time

Using the interventions in the current study, time to problem solve decreased overall for the three participants (see Table 4). In baseline and generalization, the students did not use any support from the intervention's technology to solve problems. In the baseline phase, without using any of the interventions, students spent little time completing the probes, ranging from 3.6 minutes to 3.8 minutes. This finding is best compared to the generalization phase, which was identical in format and approach, and exclusive of any interventions. The students' times varied greatly in this phase, ranging from 2 minutes to 10 minutes.

In the intervention, best treatment, and maintenance phases, the students were required to use the intervention technology to solve each probe. When their average time needed to complete the probes was compared across phases, overall completion time decreased for all participants from intervention to maintenance phases (see Table 4). During the intervention phase specifically, the VM+TBGO average completion time was the lowest for two of the three students (Gregory and Mark) compared to the other interventions. For all three students, time decreased from the best treatment to maintenance phases. Students were required to solve the four questions independently of any technological or researcher assistance in the generalization phase. Kristian and Mark decreased the amount of time they needed to solve these problems; Gregory increased the time required from maintenance to generalization phases.

Social Validity

When asked which intervention they liked best, Gregory and Mark preferred the VM+TBGO, and

Table 4
Average Time in Minutes Needed to Complete the Probes Within Phases

Student	Baseline	Intervention			Best Treatment[a]	Maintenance[a]	Generalization
		TBGO	VM	VM + TBGO			
Gregory	3.8	10.4	10.6	9	6.8	6.3	10
Kristian	3.6	7.2	7.8	8	8	5.7	2
Mark	3.8	10	9.2	8.2	6	6	5

Note. [a]Best treatment and maintenance varied for each student: Gregory = VM+TBGO, Kristian = VM, Mark = VM+TBGO.

Kristian preferred the VM only. Gregory and Mark preferred the VM+TBGO because of the blocks that were used to represent the ratios, the use of the organizer to "keep track of the blocks," and the "advanced" nature of the VM+TBGO. Kristian liked the VM only because he described it as the "quickest" intervention, and he liked using the blocks because they did not have numbers, which distracted him. Overall, the students' best treatment and maintenance intervention aligned with their preferred intervention. The students' highest scores across the interventions for accuracy and independence aligned with their preferred intervention, as stated in these interviews.

Discussion

The current study investigated the functional relation between a VM, TBGO, and a combined intervention (VM+TBGO) on accuracy in solving proportion word problems for high school students with MLD. For all three students, the three strategies were positive and supported their performance. Overall, student accuracy and independence in solving proportion word problems increased from baseline to intervention and were sustained during all students' best treatment and maintenance phases. For two students, accuracy and independence were highest when using VM+TBGO, and for one student, accuracy and independence were highest when using VM only. Additionally, the average time students took to solve the word problems decreased within phases, but comparisons could not be made between baseline and intervention phases. Finally, two of three students preferred using a VM+TBGO to solve word problems.

Connections to Prior Research

Our findings extend the research surrounding word problem instruction for students with disabilities. Currently, schematic diagrams (Jitendra et al., 2002, 2015) and the CRA sequence (Strickland & Maccini, 2012; Witzel, 2005) are commonly cited in mathematics education literature, especially for learners with disabilities. The current study found trends similar to the findings of Root et al. (2017), in which graphic organizers and a self-instruction sheet for solving mathematical word problems were combined to support elementary students with ASD and intellectual disabilities. The researchers noted that the combination of the two "facilitated the conceptual understanding of the action to the word problem as well as the procedures to follow with the manipulatives in order to arrive at the solution similar to the way a mnemonic has been used in prior schema-based instruction research" (Root et al., 2017, p. 50). By combining a VM with a TBGO, as in the current study, students integrated the use of manipulatives with a graphic organizer, which potentially facilitated the procedural knowledge and efficiency required to solve proportion word problems accurately.

As in a study by Suh and Moyer (2007), the current study provided students with the manipulative in unison with a framework in which to work, as the word problems presented to students may have been too complex to use a VM in isolation. Our findings support this assumption: Two out of the three students were more successful in solving word problems when they were given the support of the VM+TBGO application. The third student was most successful with the use of the VM only. No students preferred the TBGO, and two of three were not as successful with the TBGO in isolation. The VM+TBGO provided structured support in which students could use the VM to represent the important elements of the proportion word problems.

Additionally, this study contributes to the overall research base on VMs and how they are best used to facilitate instruction. The VRA sequence (Bouck et al., 2017) requires learners to use VMs to represent mathematics concepts and move to a representational phase and then an abstract phase. The current study followed a VA sequence, as no representations were required by the student participants. However, the two students who scored highest in the VM+TBGO intervention without being prompted drew representational drawings of the VM blocks on their paper in the generalization probe. Although this behavior only occurred in the one generalization probe, the students who completed the greatest number of probes when using the VM+TBGO drew representations of the VM while problem-solving. Such findings provide valuable insights into the significance of representational drawings and their presence – or lack thereof – within the VRA and VA teaching sequences.

Implications for Practice

Findings from this study have implications for selecting technology in mathematics classrooms. Students may have preferences when using technology in the classroom, and teachers should be cognizant of such preferences when integrating technology into their instruction. In this study, Gregory and Mark preferred and scored the highest with the combined VM+TBGO tool when answering proportion word problems. However, Kristian performed better in the VM and preferred that intervention over the others. He stated that this was due to the time it took him to complete word problems with the VM+TBGO and that he was more accurate with the VM. Thus, our findings suggest that when students use the technology they prefer, they perform accurately, independently, and in less time.

Although the size and scope of this study limits generalizability to all secondary students with MLD, the study provides further discussion as to how VMs, a widely researched intervention for students with disabilities, can be used to teach more complex mathematics content such as word problems and proportional reasoning. Using a VM in a structured manner or in conjunction with a TBGO makes it possible for students to access word problems of a more complex nature while still using the important features of a VM. Additionally, as the use of technology in the classroom and virtual learning has become more prevalent, technology-based interventions with an evidence base are increasingly more important to educators.

Limitations

The current study suggests that the VM+TBGO can benefit secondary students with MLD to solve proportion word problems. However, several limitations need mention. First, the baseline sessions did not continue during the intervention phase. Therefore, it is unclear if there were any multi-treatment or carry-over effects as the interventions were alternated and presented to the student (Ledford & Gast, 2018). Second, there was notable variability in Gregory's accuracy data. The data can appear variable and unstable because students could only score a zero, 25, 50, 75, or 100% in a probe with four problems. For example, Gregory's unstable baseline data were not addressed due to the need to move from baseline to intervention as soon as possible due to pending attendance issues in school. Third, there was only one generalization data point. According to Kratochwill et al. (2010), at least three probes must be given in a phase of an SSD to demonstrate experimental control. However, although definitive conclusions about a student's ability to solve proportion word problems without any interventions cannot be made, each student completed the generalization probe independently and with 75-100% accuracy.

Finally, the design of the probes and the combination of a lesson featuring explicit instruction with the use of the interventions may also be a limitation. The time required to solve word problems limited the number of questions per probe to four. This allowed the student to fulfill many sessions with the researcher, but it may not accurately account for their understanding of proportion word problems. Also, the two types of proportion word problems used (i.e., finding the missing quantity and finding the total) are not representative of the multitude of proportion word problem schemas that a student may encounter in this area of math. It is possible that the immediacy of effect is a result of the explicit instruction lesson rather than the interventions because students were taught how to solve problems before the intervention phase in the explicit instruction lesson. However, a lesson featuring explicit instruction mirrors those used in other SSD studies of the functional relation between the use of a VM and student performance in mathematics (e.g., Satsangi et al., 2018).

This study was limited to measuring procedural knowledge to solve proportion word problems rather than conceptual knowledge. As such, it assessed the accuracy, independence, and time required to solve each problem. Future research to gauge conceptual understanding should include measuring student understanding of the underlying concepts and relationships.

Lastly, the study was implemented one-on-one with the lead researcher as an interventionist. Future research should include teacher practitioners as interventionists. Moreover, because one-on-one instruction with students is not always practical for teachers, further research is required to determine the feasibility of using the VM+TBGO intervention in larger classroom settings.

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